Some issues of charm physics at the τ-charm factory

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Abstract

A project of a new e^+e^- collider, called Electron-Positron super c-tau factory, is being developed in Budker Institute of Nuclear Physics (Novosibirsk). It will operate at total energies from 2 to 5 GeV with unprecedented high luminosity of 10^{35} cm^{-2} sec^{-1}. A project is aiming to study τ-decays, the decays of D-mesons and Λ_c-baryon, and explore new exotic states like $Z_c(3900)$, etc.

In this short report I will shortly discuss some issues of charm physics which would be realized at the c-tau factory.

I. FOUR-QUARK STRUCTURE OF $Z_C(3900)$ STATE

The process $e^+e^- \to \pi^+\pi^- J/\psi$ has been studied by the BESIII Collaboration [1]. A structure was observed at around 3.9 GeV in the $\pi^\pm J/\psi$ mass spectrum which was christened the $Z_c(3900)$ state. If interpreted as a new particle, it is unusual in that it carries an electric charge and couples to charmonium. A fit to the $\pi^\pm J/\psi$ invariant mass spectrum results in a mass of $M_{Z_c} = (3899.0 \pm 3.6(\text{stat}) \pm 4.9(\text{syst}))$ MeV and a width of $\Gamma_{Z_c} = (46 \pm 10(\text{stat}) \pm 20(\text{syst}))$ MeV.

The cross section for $e^+e^- \to \pi^+\pi^- J/\psi$ between 3.8 GeV and 5.5 GeV was measured by the Belle Collaboration [2]. This measurement lead to the observation of the state $Y(4260)$, and its resonance parameters were determined. In addition, an excess of $\pi^+\pi^- J/\psi$ production around 4 GeV was observed. This feature can be described by a Breit-Wigner parameterization with properties that are consistent with the $Y(4008)$ state that was previously reported by Belle. In a study of the $Y(4260) \to \pi^+\pi^- J/\psi$ decays, a structure was observed in the $M(\pi^\pm J/\psi)$ mass spectrum with 5.2 σ significance, with mass $M = (3894.5 \pm 6.6(\text{stat}) \pm 4.5(\text{syst}))$ MeV and width $\Gamma = (63 \pm 24(\text{stat}) \pm 26(\text{syst}))$ MeV, where the errors are statistical and systematic, respectively. This structure can be interpreted as a new charged charmonium-like state.

Using 586 pb of e^+e^- annihilation data the CLEO-c detector made an analysis at $\sqrt{s} = 4170$ MeV at the peak of the charmonium resonance $\psi(4160)$. The subsequent decay $\psi(4160) \to \pi^+\pi^- J/\psi$ was analyzed [3], and the charged state $Z_c^\pm(3900)$ was observed which decays into $\pi^\pm J/\psi$ at a significance level of $> 5\,\sigma$. The value of the mass $M_{Zc} = 3886 \pm 4(\text{stat}) \pm 2(\text{syst})$ MeV and the width $\Gamma_{Z_c} = 37 \pm 4(\text{stat}) \pm 8(\text{syst})$ MeV were found to be in good agreement with the results for this resonance reported by the BES III and Belle Collaborations in the decay of the resonance $Y(4260)$. In addition CLEO-c presented the first evidence for the production of the neutral member of this isospin triplet, $Z_c^0(3900)$ decaying into $\pi^0 J/\psi$ at a $3.5\,\sigma$ significance level.

A study of the process $e^+e^- \to \pi^\pm(D\bar{D}^*)^\mp$ was reported by the BESIII Collaboration [4] at $\sqrt{s} = 4.26$ GeV using a 525 pb^{-1} data sample collected with the BESIII detector at the BEPCII storage ring. A distinct charged structure was observed in the $(D\bar{D}^*)^\mp$ invariant mass distribution. When fitted to a mass-dependent-width Breit-Wigner line shape, the pole mass and width were determined to be $M_{\text{pole}} = 3883.9 \pm 1.5(\text{stat}) \pm 4.2(\text{syst})$ MeV and

$\Gamma_{\rm pole} = 24.8 \pm 3.3({\rm stat}) \pm 11.0({\rm syst})$ MeV. The mass and width of the structure referred to as $Z_c(3885)$ are 2σ and 1σ, respectively, below those of the $Z_c(3900) \to \pi^\pm J/\psi$ peak observed by BESIII and Belle in $\pi^+\pi^- J/\psi$ final states produced at the same center-of-mass energy. The angular distribution of the $\pi Z_c(3885)$ system favors a $J^P = 1^+$ quantum number assignment for the structure and disfavors the assignment 1^- or 0^-. The Born cross section times the DD^* branching fraction of the $Z_c(3885)$ is measured to be

$$\sigma\left(e^+e^- \to \pi^\pm Z_c^\mp(3885)\right) \times \mathcal{B}\left(Z_c^\mp(3885) \to (D\bar{D}^*)^\mp\right) = 83.5 \pm 6.6({\rm stat}) \pm 22.0({\rm syst})\ {\rm pb}\,. \tag{1}$$

Assuming the $Z_c(3885) \to D\bar{D}^*$ signal reported in [4] and the $Z_c(3900) \to \pi J/\psi$ signal are from the same source, the ratio of partial widths is determined as

$$\frac{\Gamma(Z_c(3885) \to D\bar{D}^*)}{\Gamma(Z_c(3885) \to \pi J/\psi)} = 6.2 \pm 1.1({\rm stat}) \pm 2.7({\rm syst})\,. \tag{2}$$

That means that the $Z_c(3900)$ state has a much stronger coupling to DD^* than to $\pi J/\psi$ [5]. An unbinned maximum likehood fit gives a mass of $M = 3889.1 \pm 1.8$ MeV and a width of $\Gamma = 28.1 \pm 4.1$ MeV ($M = 3891.8 \pm 1.8$ MeV and $\Gamma = 27.8 \pm 3.9$ MeV) for the two data sets, respectively. The pole position of this peak is calculated to be $M_{\rm pole} = 3883.9 \pm 1.5 \pm 4.2$ MeV and $\Gamma_{\rm pole} = 24.8 \pm 3.3 \pm 11.0$ MeV. The mass and width of the peak observed in the DD^* final state agree with that of the $Z_c(3900)$. Thus, they are quite probably the same state.

In the paper [6] we have critically checked the tetraquark picture for the $Z_c(3900)$ state by analyzing its strong decays. In our consideration we have used the covariant quark model proposed in [7] and used in Refs. [8, 9] to describe the properties of the $X(3872)$ state as a tetraquark state. First, we employ an interpretation of the $Z_c(3900)$ state as the isospin 1 one partner of the $X(3872)$ as was suggested in Refs. [10] and [11]. Then the quantum numbers for the neutral state are $I^G(J^{PC}) = 1^+(1^{+-})$. Accordingly the interpolating current for the $Z_c^+(3900)$ state is given by:

$$J^\mu = \frac{i}{\sqrt{2}}\varepsilon_{abc}\varepsilon_{dec}\left[(u_a^T C\gamma_5 c_b)(\bar{d}_d\gamma^\mu C\bar{c}_e^T) - (u_a^T C\gamma^\mu c_b)(\bar{d}_d\gamma_5 C\bar{c}_e^T)\right] \tag{3}$$

We employ a charge conjugation matrix in the form of $C = \gamma^0\gamma^2$, i.e. without a factor "i" as is usually employed.

We have calculated the partial widths of the decays $Z_c^+(3900) \to J/\psi\pi^+$, $\eta_c\rho^+$ and $\bar{D}^0 D^{*+}$, $\bar{D}^{*0}D^+$. We found that for a relatively small model size parameter $\Lambda_{Z_c} \sim 2.25$ GeV one can reproduce the central values for the partial widths of the decays $Z_c^+ \to J/\psi\pi^+$, $\eta_c\rho^+$

as they were also obtained in Refs. [10, 11]. It turns out that, in our model, the leading Lorentz metric structure in the matrix elements describing the decays $Z_c^($3900) $\to \bar{D}D^*$ vanishes analytically. This results in a significant suppression of these decay widths by the smallness of the relevant phase space factor $|\mathbf{q}|^5$. If the parameter Λ_{Z_c} is varied in the region $\Lambda_{Z_c} = 2.25 \pm 0.10$ GeV the numerical values of the decay widths vary as

$$\begin{aligned}
\Gamma(Z_c^+ \to J/\psi + \pi^+) &= (27.9^{+6.3}_{-5.0})\,\mathrm{MeV}\,,\\
\Gamma(Z_c^+ \to \eta_c + \rho^+) &= (35.7^{+6.3}_{-5.2})\,\mathrm{MeV}\,,\\
\Gamma(Z_c^+ \to \bar{D}^0 + D^{*\,+}) &\propto 10^{-8}\,\mathrm{MeV}\,,\\
\Gamma(Z_c^+ \to \bar{D}^{*\,0} + D^+) &\propto 10^{-8}\,\mathrm{MeV}\,.
\end{aligned} \tag{4}$$

Since the experimental data [4] show that the $Z_c(3900)$ has a much more stronger coupling to DD^* than to $J/\psi\pi$, one has to conclude that the tetraquark-type current for the $Z_c(3900)$ is in discord with experiment.

As an alternative we have employed a molecular-type four-quark current to describe the decays of the $Z_c(3900)$ state as the charged particle in the isotriplet (see Ref. [12])

$$J^\mu = \frac{1}{\sqrt{2}}\left[(\bar{d}\gamma_5 c)(\bar{c}\gamma^\mu u) + (\bar{d}\gamma^\mu c)(\bar{c}\gamma_5 u)\right]. \tag{5}$$

As a guide to adjust the parameter Λ_{Z_c} we take the experimental values for decay widths given in Ref. [4]. If the parameter Λ_{Z_c} is varied in the limits $\Lambda_{Z_c} = 3.3 \pm 0.1$ GeV the numerical values of decay widths vary according to

$$\begin{aligned}
\Gamma(Z_c^+ \to J/\psi + \pi^+) &= (1.8 \pm 0.3)\,\mathrm{MeV}\,,\\
\Gamma(Z_c^+ \to \eta_c + \rho^+) &= (3.2^{+0.5}_{-0.4})\,\mathrm{MeV}\,,\\
\Gamma(Z_c^+ \to \bar{D}^0 + D^{*\,+}) &= (10.0^{+1.7}_{-1.4})\,\mathrm{MeV}\,,\\
\Gamma(Z_c^+ \to \bar{D}^{*\,0} + D^+) &= (9.0^{+1.6}_{-1.3})\,\mathrm{MeV}\,.
\end{aligned} \tag{6}$$

Thus a molecular-type current for the vertex function of the Z_c is in accordance with the experimental observation [4] that $Z_c(3900)$ has a much more stronger coupling to DD^* than to $J/\psi\pi$.

II. SEMILEPTONIC DECAYS $\Lambda_C^+ \to \Lambda\ell^+\nu_\ell$ ($\ell^+ = E^+, \mu^+$)

We have done precise theoretical predictions for the absolute branching fractions for $\Lambda_c^+ \to \Lambda\ell^+\nu_\ell\,(\ell^+ = e^+, \mu^+)$ decays in the covariant confined quark model [13]. This study was motivated by two recent precise experiments performed by the Belle Collaboration at the KEKB and by the BESIII Collaboration at the BEPCII on the first meauserements of the absolute branching fraction of $\Lambda_c^+ \to pK^-\pi^+$ and $\Lambda_c^+ \to \Lambda e^+\nu_e$.

In 2013 the Belle Collaboration at KEKB [14] reported on the first model-independent measurement of the branching fraction $\mathrm{Br}(\Lambda_c^+ \to pK^-\pi^+) = (6.84 \pm 0.24^{+0.21}_{-0.27})\%$. This measurement significantly improved the precision of the absolute branching fractions of other Λ_c^+ decay modes and of b-flavored hadrons involving the Λ_c^+ state. In particular, using the Belle result the Particle Data Group [15] updated their average for the branching fractions of the exclusive semileptonic modes of the Λ_c^+ to

$$\begin{aligned} \mathrm{Br}(\Lambda_c^+ \to \Lambda e^+\nu_e) &= (2.9 \pm 0.5)\%\,, \\ \mathrm{Br}(\Lambda_c^+ \to \Lambda \mu^+\nu_\mu) &= (2.7 \pm 0.6)\%\,. \end{aligned} \tag{7}$$

A few weeks ago the BESIII Collaboration reported on the first absolute measurent of the branching ratio of $\Lambda_c^+ \to \Lambda e^+\nu_e = (3.63 \pm 0.38(\mathrm{stat}) \pm 0.20(\mathrm{syst}))\%$ [17]. One can see that the current upper limit of the Particle Data agrees with the lower limit of the BESIII result. The new data calls for a detailed theoretical analysis of the $\Lambda_c^+ \to \Lambda \ell^+\nu_\ell$ $(\ell = e, \mu)$ process.

Below we present our predictions for the semileptonic branching ratios of the Λ_c and compare them with data from Belle [14] and BESIII [17] Collaborations. We have used the value for the Λ_c–lifetime from the Particle Data Group [15] $\tau_{\Lambda_c} = (2.0 \pm 0.06) \times 10^{-13}$ s. One can see that our results are in a good agreement with Belle data and close to lower value of the BESIII result. Also below in Eq. (9) we compare our predictions with previous theoretical results for $\Lambda_c \to \Lambda \ell \nu_\ell$ at zero charged lepton mass. For some approaches in brackets we indicate the result with taking into account $SU(6)$ spin-flavor suppression factor equal to 1/3 (see detailed discussion in Ref. [18]).

Mode	Our results	Data
$\Lambda_c^+ \to \Lambda^0 e^+\nu_e$	2.78	(2.9 ± 0.5) Belle
		$(3.63 \pm 0.38 \pm 0.20)$ BESIII
$\Lambda_c^+ \to \Lambda^0 \mu^+\nu_\mu$	2.69	(2.7 ± 0.6) Belle

(8)

Our	[19]	[20]	[21]	[21]	[22]	[23]	[24]	[25]	[26]	[27]
2.78	12(4)	3(1)	3.4(1.12)	2.6(0.86)	2	4.4(1.46)	1.42	1.07	1.44	1.4

(9)

We list the partial and total rates in units of 10^{-15} GeV. One has

$$
\begin{array}{c|ccccc|c}
 & \Gamma_U & \Gamma_L & \widetilde{\Gamma}_U & \widetilde{\Gamma}_L & 3\widetilde{\Gamma}_S & \Gamma_{\rm tot} \\
\hline
e^+\nu_e & 35.6 & 55.8 & - & - & - & 91.4 \\
\mu^+\nu_\mu & 34.3 & 50.3 & 0.3 & 0.9 & 2.8 & 88.6
\end{array}
\qquad (10)
$$

The numbers show that the partial flip rates make up 34.2 % of the total rate where the biggest contribution comes from the scalar rate with 20.4 %.

$$
\begin{array}{cccccccc}
 & < A^\ell_{FB} > & < C_F > & < P^h_z > & < P^h_x > & < P^\ell_z > & < P^\ell_x > & < \gamma > \\
\hline
e^+\nu_e & -0.21 & -0.62 & -0.87 & -0.32 & 1.00 & -0.001 & -0.25 \\
\mu^+\nu_\mu & -0.24 & -0.54 & -0.87 & -0.33 & 0.91 & -0.18 & -0.26
\end{array}
\qquad (11)
$$

When calculating the q^2–averages one has to remember to include the q^2–dependent factor $(q^2 - m_\ell^2)^2|\mathbf{p_2}|/q^2$ in the numerator and denominator of the relevant asymmetry expressions. In most of the shown cases the mean values change considerably when going from the e– to the μ–modes including even a sign change in $< A^\ell_{FB} >$.

III. DECAY CHAIN OF DOUBLE CHARM BARYON STATE Ξ^{++}_{CC}

Recently the LHCb Collaboration has reported on the discovery of the double charm state Ξ^{++}_{cc} [28] found in the invariant mass spectrum of the final state particles $(\Lambda_c^+ K^- \pi^+ \pi^+)$ where the Λ_c^+ baryon was reconstructed in the decay mode $pK^-\pi^+$. The mass of the new state was given as $3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV. The central value of the extracted mass is very close to the 3610 MeV value predicted in Ref. [18] in the framework of the one gluon exchange modelof de Rujula, Georgi and Glashow [29] which features a Breit-Fermi spin-spin interaction term. It is noteworthy that Ebert et al. predicted a mass of 3620 MeV for the Ξ^{++}_{cc} using a relativistic quark-diquark potential model [30]. We have interpreted the new double charm baryon state found in the $(\Lambda_c^+ K^- \pi^+ \pi^+)$ mass distribution as being at the origin of the decay chain $\Xi^{++}_{cc} \to \Sigma_c^{++}(\to \Lambda_c^+\pi^+) + \bar{K}^{*0}(\to K^-\pi^+)$ [31]. This decay chain is favored from an experimental point of view since the branching ratios of the daughter particle decays $\Sigma_c^{++} \to \Lambda_c^+\pi^+$ and $\bar{K}^{*0} \to K^-\pi^+$ are large ($\sim 100\,\%$ and, from isospin invariance, $\sim 66\,\%$, respectively).

The nonleptonic decay $\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^{*0}$ belongs to a class of decays where the quark flavor composition is such that the decay proceeds solely via the factorizing contribution precluding a contamination from internal W-exchange. We have used our covariant confined quark model to calculate the four helicity amplitudes that describe the dynamics of the transition $\Xi_{cc}^{++} \to \Sigma_c^{++}$ induced by the effective $(c \to u)$ current. We have then proceeded to calculate the rate of the decay as well as the polarization of the Σ_c^{++} and Λ_c^{+} baryons and the longitudinal/transverse composition of the $\bar{K}^{*0}$. The nontrivial helicity composition of the $\bar{K}^{*0}$ leads to a nontrivial angular decay distribution in terms of the polar angle θ_V formed by the direction of the K^- in the $\bar{K}^{*0}$ rest system and the original flight direction of the $\bar{K}^{*0}$.

The partial decay widths were found to be equal to

$$\Gamma(\Xi_{cc}^{++} \to \Sigma_c^{++} \, + \, \bar{K}^{*0}) = (0.21 \pm 0.02) \times 10^{12}\,\mathrm{s}^{-1}\,. \tag{12}$$

We have also analyzed the decay $\Xi_{cc}^{++} \to \Sigma_c^{++} \, + \, \bar{K}^{0}$ using the same dynamics as for the decay $\Xi_{cc}^{++} \to \Sigma_c^{++} \, + \, \bar{K}^{*0}$. We have obtained

$$\Gamma(\Xi_{cc}^{++} \to \Sigma_c^{++} \, + \, \bar{K}^{0}) = (0.05 \pm 0.01) \times 10^{12}\,\mathrm{s}^{-1}\,. \tag{13}$$

The $\bar{K}^{*0}$ mode is about four times stronger than the one including the $\bar{K}^{0}$. In order to convert the partial rate into a branching ratio one would need the total width or, equivalently, the lifetime value of the Ξ_{cc}^{++}. Neither of these are known experimentally. There have been several attempts to calculate the lifetime of the Ξ_{cc}^{++} based on the optical theorem for the inclusive decay width combined with the Operator Product Expansion for the transition currents together with a heavy quark mass expansion. The results are in the range of $430\,\mathrm{fs} - 670\,\mathrm{fs}$ [32, 34]. As a median value we take $\tau_{\Xi_{cc}^{++}} = 500\,\mathrm{fs}$. For the branching ratios we obtain

$$\begin{aligned} B(\Xi_{cc}^{++} \to \Sigma_c^{++} \, + \, \bar{K}^{*0}) &= \left(\frac{\tau_{\Xi_{cc}^{++}}}{500\,\mathrm{fs}}\right) \cdot (10.5 \pm 1)\,\%\,, \\ B(\Xi_{cc}^{++} \to \Sigma_c^{++} \, + \, \bar{K}^{0}) &= \left(\frac{\tau_{\Xi_{cc}^{++}}}{500\,\mathrm{fs}}\right) \cdot (2.5 \pm 0.5)\,\%\,. \end{aligned}$$

Now let us treat the decaying Ξ_{cc}^{++} as being unpolarized. In principle, the Ξ_{cc}^{++} could acquire a nonzero transverse polarization in the hadronic production process. However, since one is averaging over the rapidities of the production process the Ξ_{cc}^{++} is effectively unpolarized (for more details see [35]). The baryon-side decay $\Sigma_c^{++} \to \Lambda_c^{+} \, \pi^{+}$ is a strong decay and, even though the Σ_c^{++} is polarized, the decay $\Sigma_c^{++} \to \Lambda_c^+ \, \pi^+$ possesses zero analyzing power to resolve the polarization of the Σ_c^{++}, i.e. the azimuthal angle and the helicity angle decay distribution of the decay $\Sigma_c^{++} \to \Lambda_c^+\pi^+$ is uniform. For the meson-side decay $\bar{K}^{*0} \to K^-\pi^+$ one obtains the angular decay distribution

$$\begin{aligned}
\frac{d\,\Gamma(\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^{*0}(\to K^-\pi^+))}{d\cos\theta_V} &= B(\bar{K}^{*0} \to K^-\pi^+)\frac{G_F^2}{32\pi}\,\frac{|\mathbf{p_2}|}{M_1^2}\,|V_{ij}V_{kl}^*|^2\,C_{\rm eff}^2\,f_V^2\,M_V^2\;\mathcal{H}_N \\
&\times\;\left(\frac{3}{2}\cos^2\theta_V\,\mathcal{F}_L + \frac{3}{4}\sin^2\theta_V\,\mathcal{F}_T\right) \qquad (14)
\end{aligned}$$

where $B(\bar{K}^{*0} \to K^-\pi^+) = 2/3$ is the branching ratio of the decay $\bar{K}^{*0} \to K^-\pi^+$. The angular decay distribution (14) involves the helicity fractions of the $\bar{K}^{*0}$ defined by

$$\mathcal{F}_L = \frac{|H_{\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2}{\mathcal{H}_N} = 0.48 \pm 0.01\,, \qquad \mathcal{F}_T = \frac{|H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2}{\mathcal{H}_N} = 0.52 \pm 0.01\,. \quad (15)$$

This has to be compared to the unpolarized case $\mathcal{F}_L = 1/3$ and $\mathcal{F}_T = 2/3$ which is e.g. realized at the zero recoil point $q^2 = (M_1 - M_2)^2$ where there is only the axial vector S-wave excitation of the final $(\Sigma_c^{++}\,\bar{K}^{*0})$–state with $\sqrt{2}H^A_{1/2\,0} = H^A_{1/2\,1}$ ("allowed Fermi–Teller transition"). Our results for the helicity fractions considerably deviate from their unpolarized values leading to a pronounced $\cos\theta_V$–dependence of the angular decay distribution (14) which is quite close to $W(\theta_V) \sim 3/8(1+\cos^2\theta_V)$.

The longitudinal polarization of the daughter baryon Σ_c^{++} depends on the polar emission angle θ_V via

$$P_{\Sigma_c^{++}}(\cos\theta_V) = \frac{\frac{3}{4}\sin^2\theta_V\left(|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2\right) + \frac{3}{2}\cos^2\theta_V\left(|H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2\right)}{\frac{3}{4}\sin^2\theta_V\left(|H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2\right) + \frac{3}{2}\cos^2\theta_V\left(|H_{\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2\right)}\,. \qquad (16)$$

When averaged over $\cos\theta_V$ (one has to integrate the numerator and denominator separately) one has

$$P_{\Sigma_c^{++}} = \frac{\left(|H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2\right) + \left(|H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2\right)}{\mathcal{H}_N} = -(0.83 \pm 0.01)\,. \qquad (17)$$

As mentioned before the polarization of the Σ_c^{++} is not measurable in its strong decays. However, the Σ_c^{++} transfers its polarization to the Λ_c^+ in the strong decay $\Sigma_c^{++} \to \Lambda_c^+\,\pi^+$. The average longitudinal polarization of the Λ_c^+ can be calculated to be (we average over $\cos\theta_V$):

$$P_{\Lambda_c^+}(\theta_B) = \frac{|H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2 + |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2}{\mathcal{H}_N}\;\cos\theta_B \;=\; -(0.83 \pm 0.01)\;\cos\theta_B \qquad (18)$$

where θ_B is the angle between the direction of the Λ_c^+ and the original flight direction of the Σ_c^{++}, all in the rest frame of the Σ_c^{++}.

For the decay $\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^0$ we find a slightly larger value of the longitudinal polarization of the Σ_c^{++} given by

$$P_{\Sigma_c^{++}}(\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^0) = \frac{|H_{\frac{1}{2}t}|^2 - |H_{-\frac{1}{2}t}|^2}{\mathcal{H}_S} = -(0.95 \pm 0.02)\,. \tag{19}$$

In principle, the polarization of the Λ_c^+ can be analyzed in its weak decay $\Lambda_c^+ \to pK^-\pi^+$. For example, one could attempt to measure nonvanishing values of the expectation value $\langle\cos\theta_i\rangle$ where θ_i is the polar angle between the polarization direction of the Λ_c^+ and either one of the three decay particles ($i = p, K^+, \pi^-$) or the normal of the decay plain (see an exemplary analysis of a weak $(1 \to 3)$–particle decay in e.g. [36]). To our knowledge the weak decay $\Lambda_c^+ \to pK^-\pi^+$ has not been completely calculated yet except for an analysis of the subchannels $\Lambda_c^+ \to p\bar{K}^{*0}$ and $\Lambda_c^+ \to \Delta^{++}K^-$ [37].

We have discussed in some detail the possibility that the new double charm state found in the invariant mass distribution of $(\Lambda_c^+\,K^-\,\pi^+\,\pi^+)$ can be attributed to the decay chain $\Xi_{cc}^{++} \to \Sigma_c^{++}(\to \Lambda_c^+\pi^+) + \bar{K}^{*0}(\to K^-\pi^+)$. The hypothesis can be tested experimentally by looking at the decay distributions of the particles involved in the cascade decay. For once one can check whether there are significant peaks at the Σ_c^{++} and $\bar{K}^{*0}$ masses in the $(\Lambda_c^+\pi^+)$ and $(K^-\pi^+)$ invariant mass distributions, respectively. If there is a significant continuum background one would have to place relevant cuts on the invariant mass distribution to obtain the appropriate cascade decay channels discussed in this paper. One can then go on and check on the angular decay distributions in the respective cascade decays which have been written down in this paper. We have also discussed the decay $\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^0$ which we predict to have a branching ratio four times smaller than that of the decay $\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^{*0}$. It would nevertheless be interesting to experimentally search for this decay mode.

It would also be worthwhile to experimentally check on further nonleptonic decay channels of the double charm state Ξ_{cc}^{++}. For once there are the decay channels $\Xi_{cc}^{++} \to \Xi_c^{*+}(\to \Xi_c^0 + \pi^+) + \pi^+(\rho^+)$ and $\Xi_{cc}^{++} \to \Xi_c^{*++}(\to \Xi_c^+ + \pi^+) + \rho^0$. Experimentally more challenging would be the decay channels $\Xi_{cc}^{++} \to \Xi_c^{\prime +}(\to \Xi_c^+ + \gamma) + \pi^+(\rho^+)$, and $\Xi_{cc}^{++} \to \Xi_c^{*++}(\to \Xi_c^+ + \pi^+) + \pi^0$ because their detection would require photon identification. The above two-body nonleptonic decay modes belong to the same class of processes as the decays $\Xi_{cc}^{++} \to \Sigma_c^{++} + \bar{K}^{*0}(\bar{K}^0)$ in that they are solely contributed to by the factorizing (or tree graph) contribution. A possible W-exchange contribution (color commensurate "C" in the terminology of [38]) is forbidden by the Körner, Pati, Woo theorem [39].

IV. THE EXCLUSIVE DECAYS $J/\psi \to D_{(S)}^{(*)-}\ell^+\nu_\ell$

Low lying states of quarkonia systems similar to J/ψ usually decay through intermediate photons or gluons produced by the parent $q\bar{q}$ quark pair annihilation [40]. As a result, strong and electromagnetic decays of J/ψ have been largely investigated while weak decays of J/ψ have been put aside for decades. However, in the last few years many improvements in instruments and experimental techniques, in particular, the luminosity of colliders, have led to observation of many rare processes including the extremely rare decays $B_{(s)}^0 \to \mu^+\mu^-$, announced lately by the CMS and LHCb collaborations [41]. The branching fractions were measured to be $\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (2.8^{+0.7}_{-0.6}) \times 10^{-9}$ and $\mathcal{B}(B^0 \to \mu^+\mu^-) = (3.9^{+1.6}_{-1.4}) \times 10^{-10}$. This raises the hope that one may also explore the rare weak decays of charmonium and draws researchers' attention back to these modes.

Recently, BESIII Collaboration reported on their search for semileptonic weak decays $J/\psi \to D_s^{(*)-}e^+\nu_e + c.c.$ [42], where "$+\,c.c.$" indicates that the signals were sum of these modes and the relevant charge conjugated ones. The results at 90% confidence level were found to be $\mathcal{B}(J/\psi \to D_s^- e^+\nu_e + c.c.) < 1.3 \times 10^{-6}$ and $\mathcal{B}(J/\psi \to D_s^{*-} e^+\nu_e + c.c.) < 1.8 \times 10^{-6}$. Although these upper limits are far above the predicted values within the Standard Model (SM), which are of the order of $10^{-8} - 10^{-10}$ [43–45], one should note that this was the first time an experimental constraint on the branching fraction $\mathcal{B}(J/\psi \to D_s^{*-} e^+\nu_e + c.c.)$ was set, and moreover, the constraint on the branching fraction $\mathcal{B}(J/\psi \to D_s^- e^+\nu_e + c.c.)$ was 30 times more stringent than the previous one [15]. With a huge data sample of 10^{10} J/ψ events accumulated each year, BESIII is expected to detect these decays, even at SM levels, in the near future.

From the theoretical point of view, these weak decays are of great importance since they may lead to better understanding of non-perturbative QCD effects taking place in transitions of heavy quarkonia. Moreover, the semileptonic modes $J/\psi \to D_{(s)}^{(*)}\ell\nu$, as three-body weak decays of a vector meson, supply plentiful information about the polarization observables that can be used to probe the hidden structure and dynamics of hadrons. Additionally, these decays may also provide some hints of new physics beyond the SM, such as TopColor models [46], the Minimal Supersymmetric Standard Model (MSSM) with or without R-parity [47], and the two-Higgs-doublet models (2HDMs) [48, 49].

The very first estimate of $\mathcal{B}(J/\psi \to D_s^{(*)}\ell\nu)$ was made based on the (approximate) spin symmetry of heavy mesons, giving an inclusive branching fraction of $(0.4 - 1.0) \times 10^{-8}$, summed over D_s, D_s^*, e, μ and both charge conjugate modes [43]. In this work the transition form factors were parameterized through a universal function, similar to the Isgur-Wise function in heavy quark limit. However, the zero-recoil approximation adopted in calculating the hadronic matrix elements led to large uncertainties in the decay widths evaluation. For that reason, author of [43] noted that these results should be viewed as an estimate suggesting experimental searching, rather than a definite prediction. Latterly, by employing QCD sum rules (QCD SR) [44] or making use of the covariant light-front quark model (LFQM) [45], new theoretical studies found the branching fractions of $J/\psi \to D_s^{(*)-}e^+\nu_e + c.c.$ to be of the order of 10^{-10}. However, the results presented in [45] were about $2-8$ times larger than those calculated in [44]. Besides, one can significantly reduce hadronic uncertainties and other physical constants like G_F and $|V_{cs}|$ by considering the ratio of branching fractions $R \equiv \mathcal{B}(J/\psi \to D_s^*\ell\nu)/\mathcal{B}(J/\psi \to D_s\ell\nu)$. This ratio had been predicted to be $\simeq 1.5$ in [43] while the recent study [44] suggested $R \simeq 3.1$. Clearly, more theoretical studies and cross-check would be necessary.

In the paper [50] we have offered an alternative approach to the investigation of the exclusive decays $J/\psi \to D_{(s)}^{(*)-}\ell^+\nu_\ell$, in which we have employed the covariant constituent quark model with built-in infrared confinement (for short: confined covariant quark model (CCQM)) as dynamical input to calculate the non-perturbative transition matrix elements.

First, we present our results for leptonic decay constants of J/ψ and $D_{(s)}^{(*)}$ mesons in Table I. We also list the values of these constants obtained from experiments or other theoretical studies for comparison. One can see that our calculated values are consistent (within 10%) with results of other studies.

We present our results for the branching fractions in Table II together with results of other theoretical studies based on QCD SR and LFQM for comparison. It is worth mentioning that all values for $\mathcal{B}(J/\psi \to D_{(s)}^*\ell\nu)$ are fully consistent with those in [44]. Regarding $\mathcal{B}(J/\psi \to D_{(s)}\ell\nu)$, our results are larger than those in [44] by a factor of $2-3$. We think this discrepancy is mainly due to the values of the meson leptonic decay constants $f_D = 166\,\mathrm{MeV}$ and $f_{D_s} = 189\,\mathrm{MeV}$ used in [44], which are much smaller than $f_D = 206.1\,\mathrm{MeV}$

TABLE I: Results for the leptonic decay constants f_H in MeV.

	This work	Other	Ref.
$f_{J/\psi}$	415.0	418±9	LAT & QCD SR [52]
f_D	206.1	204.6±5.0	PDG [15]
f_{D^*}	244.3	$245(20)^{+3}_{-2}$	LAT [53]
		$278 \pm 13 \pm 10$	LAT [54]
		$252.2 \pm 22.3 \pm 4$	QCD SR [55]
f_{D_s}	257.5	257.5±4.6	PDG [15]
$f_{D_s^*}$	272.0	$272(16)^{+3}_{-20}$	LAT [53]
		311±9	LAT [54]
		$305.5 \pm 26.8 \pm 5$	QCD SR [55]
f_{D_s}/f_D	1.249	1.258±0.038	PDG [15]
$f_{D_s^*}/f_{D^*}$	1.113	1.16±0.02±0.06	LAT [54]

TABLE II: Semileptonic decay branching fractions of J/ψ meson.

Mode	Unit	This work	QCD SR [44]	LFQM [45]
$J/\psi \to D^- e^+ \nu_e$	10^{-12}	17.1	$7.3^{+4.3}_{-2.2}$	$51-57$
$J/\psi \to D^- \mu^+ \nu_\mu$	10^{-12}	16.6	$7.1^{+4.2}_{-2.2}$	$47-55$
$J/\psi \to D_s^- e^+ \nu_e$	10^{-10}	3.3	$1.8^{+0.7}_{-0.5}$	$5.3-5.8$
$J/\psi \to D_s^- \mu^+ \nu_\mu$	10^{-10}	3.2	$1.7^{+0.7}_{-0.5}$	$5.5-5.7$
$J/\psi \to D^{*-} e^+ \nu_e$	10^{-11}	3.0	$3.7^{+1.6}_{-1.1}$	–
$J/\psi \to D^{*-} \mu^+ \nu_\mu$	10^{-11}	2.9	$3.6^{+1.6}_{-1.1}$	–
$J/\psi \to D_s^{*-} e^+ \nu_e$	10^{-10}	5.0	$5.6^{+1.6}_{-1.6}$	–
$J/\psi \to D_s^{*-} \mu^+ \nu_\mu$	10^{-10}	4.8	$5.4^{+1.6}_{-1.5}$	–

and $f_{D_s} = 257.5\,\text{MeV}$ used in our present paper. In contrast, the constants $f_{D^*} = 240\,\text{MeV}$ and $f_{D_s^*} = 262\,\text{MeV}$ used in [44] are very close to our values of $f_{D^*} = 244.3\,\text{MeV}$ and $f_{D_s^*} = 272.0\,\text{MeV}$, resulting in a full agreement in $\mathcal{B}(J/\psi \to D^*_{(s)}\ell\nu)$ between the two studies. Comparing with another study, our results for $\mathcal{B}(J/\psi \to D_{(s)}\ell\nu)$ are smaller than those in [45] by a factor of $2-3$.

It is interesting to consider the ratio $R \equiv \mathcal{B}(J/\psi \to D_s^*\ell\nu)/\mathcal{B}(J/\psi \to D_s\ell\nu)$, where a large part of theoretical and experimental uncertainties cancels. We list in (20) all available predictions for R up till now.

$$R \equiv \frac{\mathcal{B}(J/\psi \to D_s^* \ell\nu)}{\mathcal{B}(J/\psi \to D_s \ell\nu)} = \begin{cases} 1.5 & \text{M.A. Sanchis-Lonzano [43]} \\ 3.1 & \text{Y.M. Wang [44]} \\ 1.5 & \text{M.A. Ivanov [50]} \end{cases} \tag{20}$$

Wang's result for R is about two times greater than our prediction because their branching fraction $\mathcal{B}(J/\psi \to D_s \ell\nu)$ is about two times smaller than ours (mainly due to the leptonic decay constants). Therefore, we propose that the value $R \simeq 1.5$ is a reliable prediction.

Moreover, we also consider the ratios

$$R_1 \equiv \frac{\mathcal{B}(J/\psi \to D_s \ell\nu)}{\mathcal{B}(J/\psi \to D \ell\nu)} \qquad \text{and} \qquad R_2 \equiv \frac{\mathcal{B}(J/\psi \to D_s^* \ell\nu)}{\mathcal{B}(J/\psi \to D^* \ell\nu)}\,, \tag{21}$$

which should be equal to $\dfrac{|V_{cs}|^2}{|V_{cd}|^2} \simeq 18.4$ under $SU(3)$ flavor symmetry limit. These ratios are $R_1 \simeq 24.7$ and $R_2 \simeq 15.1$ in [44]. We have the following values $R_1 \simeq 19.3$ and $R_2 \simeq 16.6$, which suggest a relative small $SU(3)$ symmetry breaking effect.

V. DECAY $D \to K^{(*)}\ell^+\nu_\ell$

The semileptonic decays involve strong as well as weak interactions. The extraction of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements from these exclusive decays can be parameterized by form factor calculations. As $|V_{cd}|$ and $|V_{cs}|$ are constrained by CKM unitarity, the calculation of semileptonic decays of D-mesons can also be an important test to look for new physics.

The decay $D \to K^{(*)}\ell^+\nu_\ell$ provides accurate determination of $|V_{cs}|$. Thus, the theoretical prediction for the form factors and their q^2-dependence need to be tested. A comprehensive review of experimental and theoretical challenges in study of hadronic decays of D and D_s mesons along with required experimental and theoretical tools [56] provide motivation to look into semileptonic decays.

Recently, BESIII [57–60] and *BABAR* [61] collaborations have reported precise and improved measurements on semileptonic form factors and branching fractions on decays of $D \to K\ell^+\nu_\ell$ and $D \to \pi\ell^+\nu_\ell$. A brief review of the earlier work and present experimental status of D-meson decays are given in [62]. Also there are variety of theoretical models available in the literature for the computation of hadronic form factors. One of the oldest model is based on the quark model known as ISGW model for CP violation in semileptonic B meson decays based on the nonrelativistic constituent quark picture [63]. The advanced version (ISGW2 model [64]) includes the heavy quark symmetry and has been used for semileptonic decays of $B_{(s)}$, $D_{(s)}$ and B_c mesons. The form factors are also calculated in Lattice Quantum Chromodynamics (LQCD) [65–70], light-cone sum rules (LCSR) [71–73] and LCSR with heavy quark effective theory [74]. The form factor calculations from LCSR provide good results at low ($q^2 \simeq 0$) and high ($q^2 \simeq q^2_{max}$) momentum transfers. The form factors have also been calculated for the process $D \to K\ell\nu_\ell$ in the entire momentum transfer range [70] using the LQCD. Also recently the Flavour Lattice Averaging Group (FLAG) have reported the latest lattice results for determination of CKM matrices within the standard model[75].

The form factors of $D, B \to P, V, S$ transitions with P, V and S corresponding to pseudoscalar, vector and scalar meson respectively have been evaluated in the light front quark model (LFQM) [76]. The form factors for $D \to P, V$ are also computed in the framework of chiral quark model (χQM) [77] as well in the phenomenological model based on heavy meson chiral theory (HMχT) [78, 79]. The form factors of $B_{(s)}, D_{(s)} \to \pi, K, \eta$ have been evaluated in three flavor hard pion chiral perturbation theory [80]. The form factors for $D \to \pi e^+\nu_e$ have been computed in the framework of "charm-changing current" [81]. The authors of [82, 83] have determined the form factors $f_+^{K(\pi)}$ by globally analysing the available measurements of branching fractions for $D \to K(\pi)e^+\nu_e$. The vector form factors for $D \to K\ell\nu_\ell$ were also parameterized in [84]. The evaluation of transition form factors and decays of $B_{(s)}, D_{(s)} \to f_0(980), K_0^*(1430)\ell\nu_\ell$ has been done in [85, 86] from QCD sum rules. The computation of differential branching fractions for $D_{(s)} \to (P, V, S)\ell\nu_\ell$ was also performed using chiral unitary approach [87, 88], generalized linear sigma model [89, 90] and sum rules [91]. Various decay properties of $D_{(s)}$ and $B_{(s)}$ are also studied in the formalism of semi-relativistic [92–95] and relativistic [96–98] potential models.

In the paper [99], the covariant constituent quark model (CQM) has been employed to compute the leptonic and semileptonic decays. The form factors of these transitions are expressed through only few universal functions. One of the key feature of CQM is access to the entire physical range of momentum transfer.

The resultant branching fractions for $\ell = \tau, \mu$ and e are given in Table III. It is important to note that the helicity flip factor $(1 - m_\ell^2/m_D^2)$ affects the leptonic branching fractions because of the different lepton masses. The branching fraction for $D^+ \to \mu^+\nu_\mu$ shows very good agreement with BESIII [102] and CLEO-c [103] data. The branching fractions for $D^+ \to e^+\nu_e$ and $D^+ \to \tau^+\nu_\tau$ also fulfill the experimental constraints. The branching frac-

TABLE III: Leptonic D^+-decay branching fraction ($\tau_{D^+} = 1.040 \times 10^{-12}$ s [16])

Channel	Present	Data	Reference
$D^+ \to e^+\nu_e$	8.953×10^{-9}	$< 8.8 \times 10^{-6}$	PDG [16]
$D^+ \to \mu^+\nu_\mu$	3.803×10^{-4}	$(3.71 \pm 0.19) \times 10^{-4}$	BESIII [102]
		$(3.82 \pm 0.32) \times 10^{-4}$	CLEO-c [103]
$D^+ \to \tau^+\nu_\tau$	1.013×10^{-3}	$< 1.2 \times 10^{-3}$	PDG [16]

tions for $D \to K^{(*)}\ell^+\nu_\ell$ and $D \to \pi\ell^+\nu_\ell$ are presented in Table IV. The obtained results are compared with experimental data. The results for $\mathcal{B}(D^+ \to \overline{K}^0\ell^+\nu_\ell)$ and $\mathcal{B}(D^0 \to K^-\ell^+\nu_\ell)$, ($\ell = e$ and μ) show excellent agreement with the recent BESIII data [57–59] as well with the other experimental collaborations. Also the ratios of the different semileptonic decay widths for the channels $D \to K\ell^+\nu_\ell$ are presented in Table V and the obtained results are well within the isospin conservation rules given in Ref. [104]. The results for $\mathcal{B}(D^0 \to K^*(892)^-\ell^+\nu_e)$ are also given but they overestimate the data given in PDG [16]. This deviation of the present study within the Standard Model might be explained through hadronic uncertainty or ratios of differential distributions for longitudinal and transverse polarizations of these K^* mesons [105]. The FOCUS [106] and CLEO-c [107] experiments have also reported mixing of scalar amplitudes with dominant vector decays. These observations open up new possibilities of investigations in charm semileptonic decays. There have also been attempts to explain these exclusive decays using R-parity violating supersymmetric effects [108] and their direct correlation with possible supersymmetric signals expected from LHC and BESIII data. We predict the branching fractions for $D^+ \to \overline{K}^*(892)^0\ell^+\nu_e$ but we do not compare our results since no experimental results available for this channel.

The results for branching fractions of $D^+ \to \pi^0\ell^+\nu_\ell$ and $D^0 \to \pi^-\ell^+\nu_\ell$ transitions are also given. The prediction for $\mathcal{B}(D^+ \to \pi^0 e^+\nu_e)$ is higher than BESIII [57] and CLEO-c data [100] while the trend is opposite in the case of $\mathcal{B}(D^0 \to \pi^- e^+\nu_e)$. The deviation of the $\mathcal{B}(D^+ \to \pi^0 e^+\nu_e)$ from experimental and LQCD data might be attributed to the computed form factors. However, $\mathcal{B}(D^0 \to \pi^- e^+\nu_e)$ is in close proximity to that by Belle [101] and

$\mathcal{B}(D^0 \to \pi^- \mu^+ \nu_\mu)$ is in excellent agreement with PDG data [16].

TABLE IV: Branching fractions of $D \to K^{(*)} \ell^+ \nu_\ell$ and $D \to \pi \ell^+ \nu_\ell$ (in %)

Channel	Present	Data	Reference
$D^+ \to \overline{K}^0 e^+ \nu_e$	8.84	$8.60 \pm 0.06 \pm 0.15$	BESIII [57]
		$8.83 \pm 0.10 \pm 0.20$	CLEO-c [100]
$D^+ \to \overline{K}^0 \mu^+ \nu_\mu$	8.60	$8.72 \pm 0.07 \pm 0.18$	BESIII [58]
$D^+ \to \pi^0 e^+ \nu_e$	0.619	$0.363 \pm 0.08 \pm 0.05$	BESIII [57]
		$0.405 \pm 0.016 \pm 0.009$	CLEO-c [100]
$D^+ \to \pi^0 \mu^+ \nu_\mu$	0.607	–	–
$D^+ \to \overline{K}^*(892)^0 e^+ \nu_e$	8.35	–	–
$D^+ \to \overline{K}^*(892)^0 \mu^+ \nu_\mu$	7.94	–	–
$D^0 \to K^- e^+ \nu_e$	3.46	3.538 ± 0.033	PDG [16]
		$3.505 \pm 0.014 \pm 0.033$	BESIII [59]
		$3.50 \pm 0.03 \pm 0.04$	CLEO-c [100]
		$3.45 \pm 0.07 \pm 0.20$	Belle [101]
$D^0 \to K^- \mu^+ \nu_\mu$	3.36	3.33 ± 0.13	PDG [16]
		$3.505 \pm 0.014 \pm 0.033$	BESIII
$D^0 \to \pi^- e^+ \nu_e$	0.239	$0.2770 \pm 0.0068 \pm 0.0092$	*BABAR* [61]
		$0.295 \pm 0.004 \pm 0.003$	BESIII [59]
		$0.288 \pm 0.008 \pm 0.003$	CLEO-c [100]
		$0.255 \pm 0.019 \pm 0.016$	Belle [101]
$D^0 \to \pi^- \mu^+ \nu_\mu$	0.235	0.238 ± 0.024	PDG [16]
$D^0 \to K^*(892)^- e^+ \nu_e$	3.25	2.16 ± 0.16	PDG [16]
$D^0 \to K^*(892)^- \mu^+ \nu_\mu$	3.09	1.92 ± 0.25	PDG [16]

TABLE V: Ratios of the semileptonic decays of D mesons

Ratio	Value
$\Gamma(D^0 \to K^- e^+ \nu_e)/\Gamma(D^+ \to \overline{K}^0 e^+ \nu_e)$	1.02
$\Gamma(D^0 \to K^- \mu^+ \nu_\mu)/\Gamma(D^+ \to \overline{K}^0 \mu^+ \nu_\mu)$	0.99
$\Gamma(D^+ \to \overline{K}^0 \mu^+ \nu_\mu)/\Gamma(D^+ \to \overline{K}^0 e^+ \nu_e)$	0.97

TABLE VI: Averages of forward-backward asymmetry and convexity parameters

Channel	ℓ	$\langle A^\ell_{FB} \rangle$	$\langle C^\ell_F \rangle$	$\langle C^h_F \rangle$
$D \to K$	e	-4.27 $\times$ 10^{-6}	-1.5	3
	μ	-0.058	-1.32	3
$D \to K^*$	e	0.17	-0.45	0.91
	μ	0.13	-0.37	0.89

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