

Mass spectrum and strong decays of strangeonium in a constituent quark model

<https://arxiv.org/pdf/2004.05786>

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Кварковая модель

■ Нерелятивистская кварковая модель

$$V(r) = H_0(r) + H_{sd}(r),$$

$$H_0(r) = -\frac{4}{3} \frac{\alpha_s}{r} + br + C_0$$

$$H_{sd}(r) = H_{SS} + H_T + H_{LS},$$

$$H_{SS} = \frac{32\pi\alpha_s}{9m_q m_{\bar{q}}} \tilde{\delta}_\sigma(r) \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}}$$

$$H_T = \frac{4}{3} \frac{\alpha_s}{m_q m_{\bar{q}}} \frac{1}{r^3} \left(\frac{3\mathbf{S}_q \cdot \mathbf{r} \mathbf{S}_{\bar{q}} \cdot \mathbf{r}}{r^2} - \mathbf{S}_q \cdot \mathbf{S}_{\bar{q}} \right).$$

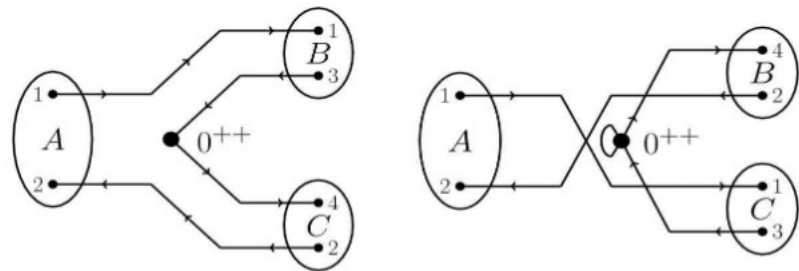
$$H_{sym} = \frac{\mathbf{S}_+ \cdot \mathbf{L}}{2} \left[\left(\frac{1}{2m_q^2} + \frac{1}{2m_{\bar{q}}^2} \right) \left(\frac{4}{3} \frac{\alpha_s}{r^3} - \frac{b}{r} \right) + \frac{8\alpha_s}{3m_q m_{\bar{q}} r^3} \right],$$

$$H_{anti} = \frac{\mathbf{S}_- \cdot \mathbf{L}}{2} \left(\frac{1}{2m_q^2} - \frac{1}{2m_{\bar{q}}^2} \right) \left(\frac{4}{3} \frac{\alpha_s}{r^3} - \frac{b}{r} \right).$$

TABLE I: The parameters of the nonrelativistic potential model.

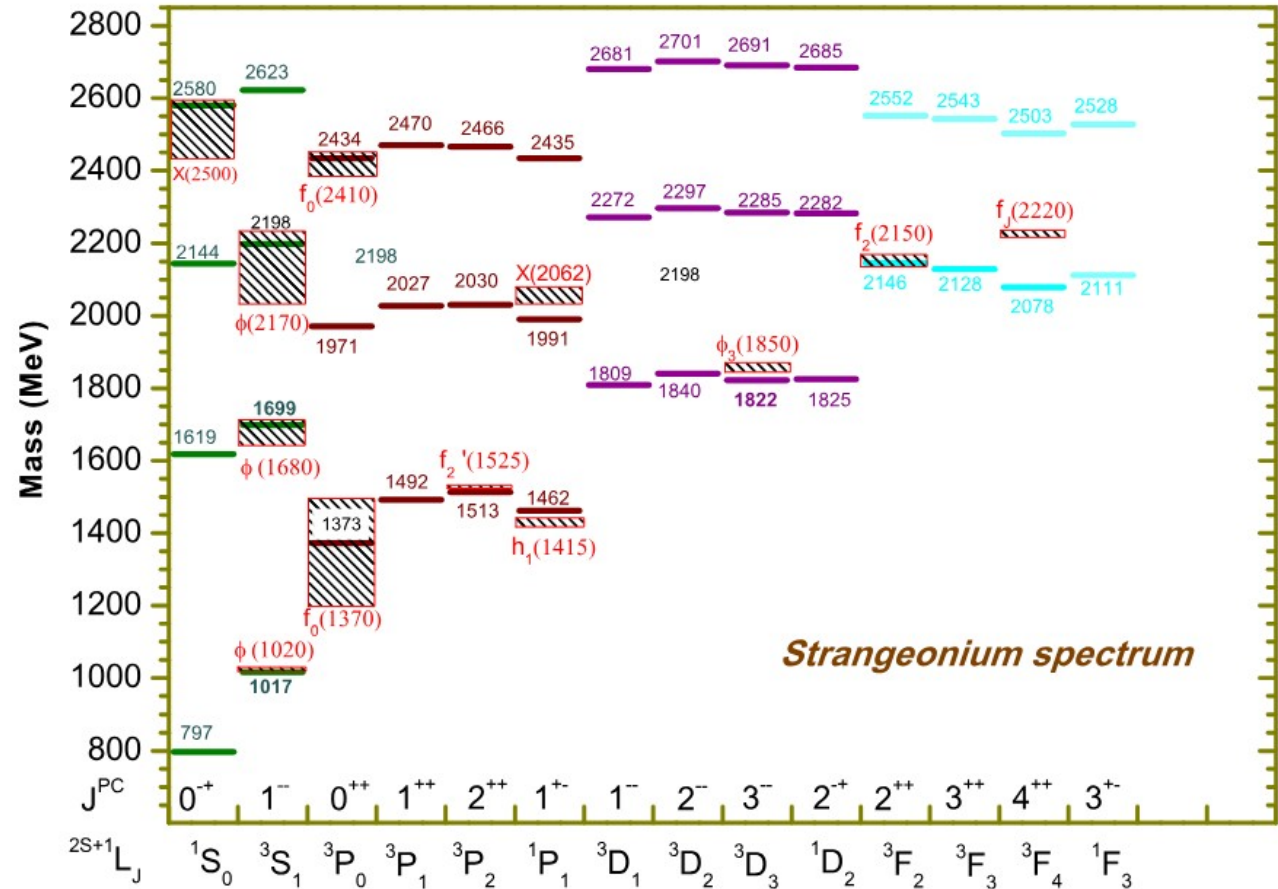
	This work	Ref. [33]
m_s (GeV)	0.600	Same
α_s	0.770	Same
σ (GeV)	0.600	Same
b (GeV ²)	0.135	0.110
C_0 (GeV)	-0.519	-0.694

3P_0 модель



Спектр странджония

- Предсказанных состояний на много больше



Спектр странджония

- Предсказанных состояний на много больше

TABLE II: Predicted masses of the $s\bar{s}$ states in this work compared with the other predictions and observations.

$n^{2S+1}L_J$	J^{PC}	Ours	XWZZ [18]	EFG [20]	SIKY [19]	GI [17]	Pang [6]	VFV [21]	Observed state [1]
1^1S_1	1^-	1017	1009	1038	1020	1020	1030	1020	$\phi(1020)$
1^1S_0	0^+	797	657	743	690	960	...	956	...
2^3S_1	1^-	1699	1688	1698	1740	1690	1687	1726	$\phi(1680)$
2^1S_0	0^+	1619	1578	1536	1440	1630	...	1795	...
3^3S_1	1^-	2198	2204	2119	2250	...	2149	...	...
3^1S_0	0^+	2144	2125	2085	1970	...	...	...	...
4^3S_1	1^-	2623	2627	2472	2540	...	2498	...	...
4^1S_0	0^+	2580	2568	2439	2260	...	...	...	$X(2500)$ [8]
1^3P_2	2^{++}	1513	1539	1529	1480	1530	...	1556	$f_2'(1525)$
1^3P_1	1^{++}	1492	1480	1464	1430	1480	...	1508	$f_1(1420)?$
1^3P_0	0^{++}	1373	1355	1420	1180	1360	...	...	$f_0(1370)$
1^1P_1	1^{+-}	1462	1473	1485	1460	1470	...	1511	$h_1(1415)$
2^3P_2	2^{++}	2030	2046	2030	2080	2040	...	1999	$f_2(2010)$
2^3P_1	1^{++}	2027	2027	2016	2020	2030	...	...	...
2^3P_0	0^{++}	1971	1986	1969	1800	1990	...	...	...
2^1P_1	1^{+-}	1991	2008	2024	2040	2010	...	1973	$X(2062)$ [5]
3^3P_2	2^{++}	2466	2480	2412	2540	...	...	...	...
3^3P_1	1^{++}	2470	2468	2403	2480	...	...	...	...
3^3P_0	0^{++}	2434	2444	2364	2280	...	...	...	$f_0(2410)$ [7]
3^1P_1	1^{+-}	2435	2449	2398	2490	...	...	...	...
1^3D_3	3^-	1822	1897	1950	1830	1900	...	1875	$\phi_3(1850)$
1^3D_2	2^-	1840	1904	1908	1810	1910	...	...	...
1^3D_1	1^-	1809	1883	1845	1750	1880	1869	...	...
1^1D_2	2^{+-}	1825	1893	1909	1830	1890	...	1853	...
2^3D_3	3^-	2285	2337	2338	2360	...	...	...	...
2^3D_2	2^-	2297	2348	2323	2330	...	...	...	...
2^3D_1	1^-	2272	2342	2258	2260	...	2276	...	...
2^1D_2	2^{+-}	2282	2336	2321	2340	...	...	...	...
3^3D_3	3^-	2691	2725	2727	...	...	...	...	...
3^3D_2	2^-	2701	2734	2667	...	...	...	...	...
3^3D_1	1^-	2681	2732	2607	...	...	2593	...	...
3^1D_2	2^{+-}	2685	2723	2662	...	...	...	...	...
1^3F_4	4^{++}	2078	2202	2286	2130	2200	...	...	$f_4(2210)$ [40]
1^3F_3	3^{++}	2128	2234	2215	2120	2230	...	...	...
1^3F_2	2^{++}	2146	2243	2143	2090	2240	...	...	$f_2(2150)$
1^1F_3	3^{+-}	2111	2223	2209	2130	2220	...	...	...
2^3F_4	4^{++}	2503	2596	2657	...	...	...	...	...
2^3F_3	3^{++}	2543	2623	2585	...	...	...	...	...
2^3F_2	2^{++}	2552	2636	2514	...	...	...	...	...
2^1F_3	3^{+-}	2528	2613	2577	...	...	...	...	...

$\varphi(1680) (2^3S_1)$

- Хорошо изучен на ВЭПП-2000

$$\Gamma_{\text{total}} = 167 \text{ MeV}$$

$$\frac{\Gamma(KK)}{\Gamma(KK^*)} \approx 0.06$$

$$R_{\eta\phi/KK^*} = \frac{\Gamma(\eta\phi)}{\Gamma(KK^*)} \approx 0.20$$

$\phi_1 (1^3D_1) 1^{--}$

- До сих пор не обнаружен

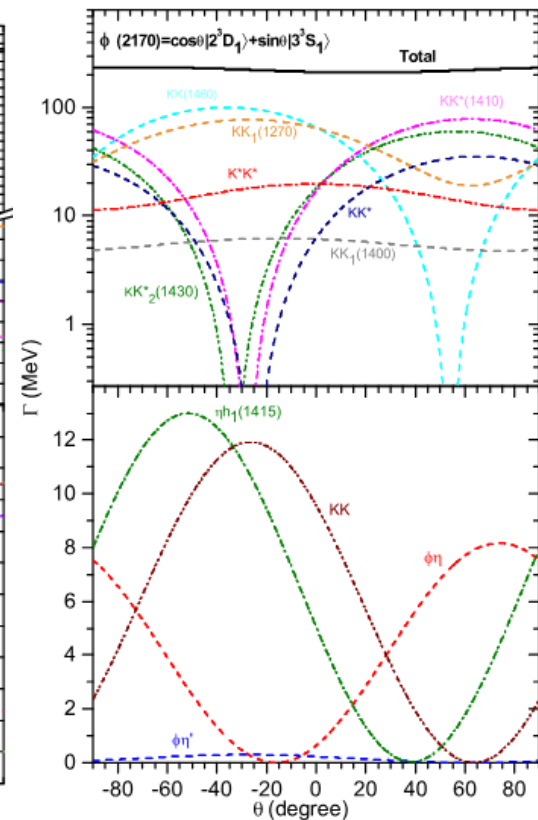
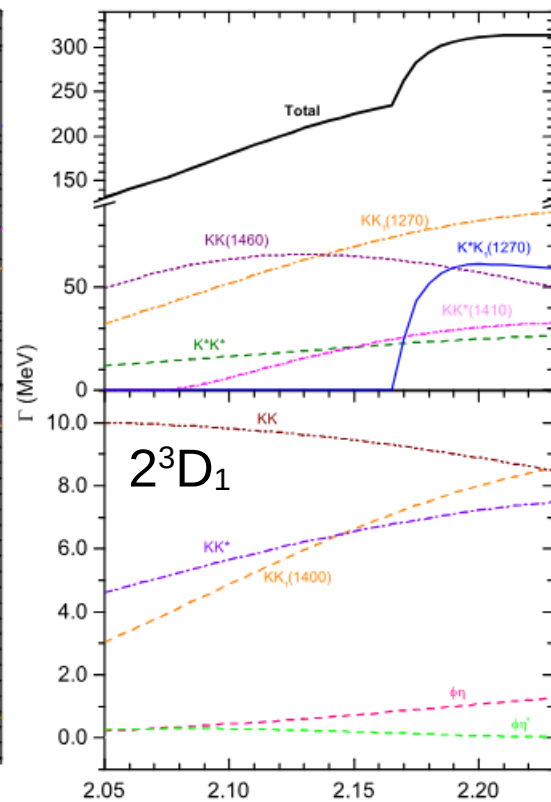
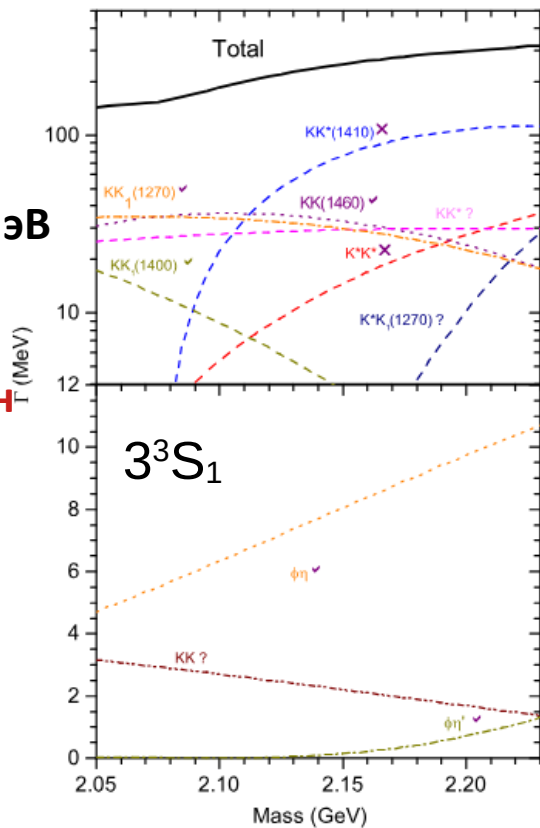
$$\Gamma_{total} \simeq 707 \text{ MeV.}$$

$$\Gamma[\phi_1(1^3D_1) \rightarrow KK_1(1270)] \simeq 620 \text{ MeV,}$$
$$Br[\phi_1(1^3D_1) \rightarrow KK_1(1270)] \simeq 88\%.$$

$\phi(2170)$

Основные моды:

- $\phi\pi^+\pi^-$, $\Gamma \cdot B_{ee} \approx 30$ эВ
- $\phi\eta'$, $\Gamma \cdot B_{ee} \approx 7$ эВ
- $\phi\eta$, $\Gamma \cdot B_{ee} \approx 0.2$ эВ
- $\phi f_0(980)$, $\Gamma \cdot B_{ee} \approx 2.5$ эВ
- $K_S K_L$, $\Gamma \cdot B_{ee} \approx 1$ эВ,
- $K^* K$, $\Gamma \cdot B_{ee} \approx 1-7$ эВ
- $K^* K^*$ не обнаружен



$f_1(1420) (1^3P_1)$

- Два кандидата $f_1(1420)$ и $f_1(1510)$ (ненадежно установлен)
- Основная мода $KK^*(892)$
- Предсказываемые моды распада сильно больше измеренных
- $J/\psi \rightarrow \gamma X$, $X \rightarrow KK^*(892)$

$f_0(1500) (1^3P_0)$

- Три кандидата $f_0(1370)$, $f_0(1500)$ и $f_0(1720)$
- Кандидаты в глюболлы
- $J/\psi \rightarrow \gamma \pi^0 \pi^0$, $\gamma K K$, $\gamma \eta \eta$

$\varphi_3 (1850) (1^3D_3)$

- Хорошо изучен в реакциях $K^- p \rightarrow K^- \bar{K} \Lambda$
- Вероятности распадов $K^* K^*$, KK^* , и KK $\sim 41\%$, $\sim 32\%$, and $\sim 23\%$,
- Важно измерить моду распада $K^* K^*$

$f'_2(1525) (1^3P_2)$

- Хорошо установленное состояние
- Возможна примесь легких кварков

$$\Gamma_{\text{total}} \simeq 58 \text{ MeV}$$

$$Br[f'_2(1525) \rightarrow KK] \simeq 70\%,$$

$$Br[f'_2(1525) \rightarrow \eta\eta] \simeq 9\%.$$

$$Br[f'_2(1525) \rightarrow KK^*(892)] \simeq 21\%.$$

$h1(1415) (1^1P_1)$

- Изучается в распадах χ_{cJ} и J/ψ
- $M = (1423.2 \pm 9.4) \text{ MeV}$ и $\Gamma = (90.3 \pm 27.3) \text{ MeV}$

$$\Gamma_{\text{total}} \simeq \Gamma[h1(1415) \rightarrow KK^*(892)] \simeq 141 \text{ MeV},$$

(11/13) Born cross-section

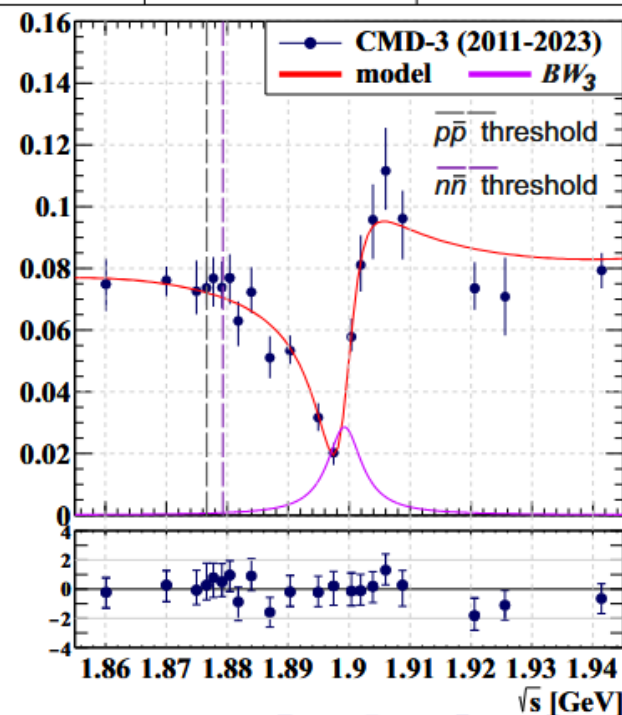
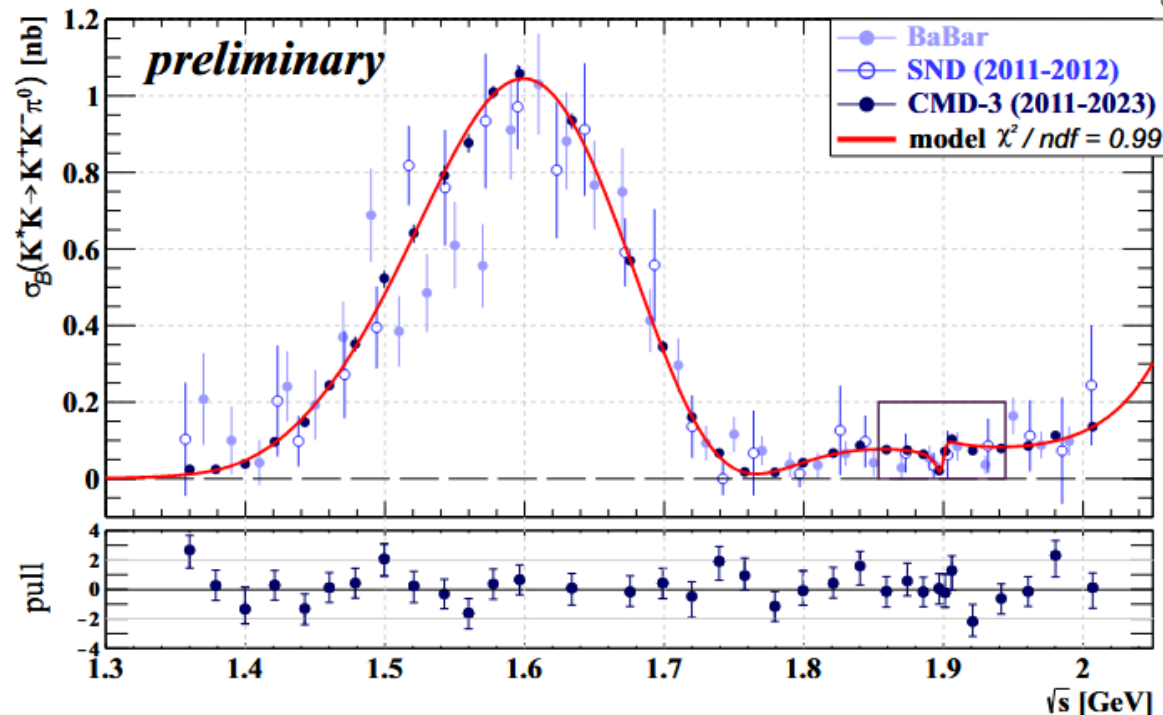
The visible cross-section fit function: $f_V(\sqrt{s}, \mathbf{p}) = \int dx f_B(\sqrt{s(1-x)}, \mathbf{p}) R_{FK}(x, s)$.
 Denotations: f_B is a function of Born cross-section model, $\mathbf{p}$ is a vector of model parameters and $R_{FK}(x, s)$ is a well-known Fadin-Kuraev kernel.

$1 + \delta_r(\sqrt{s}) = f_B(\sqrt{s})/f_V(\sqrt{s})$ – the radiative correction.

$\Rightarrow$ Born cross-section is calculated as $\sigma_B[\sqrt{s}] = (1 + \delta_r(\sqrt{s}))\sigma_V[\sqrt{s}]$.

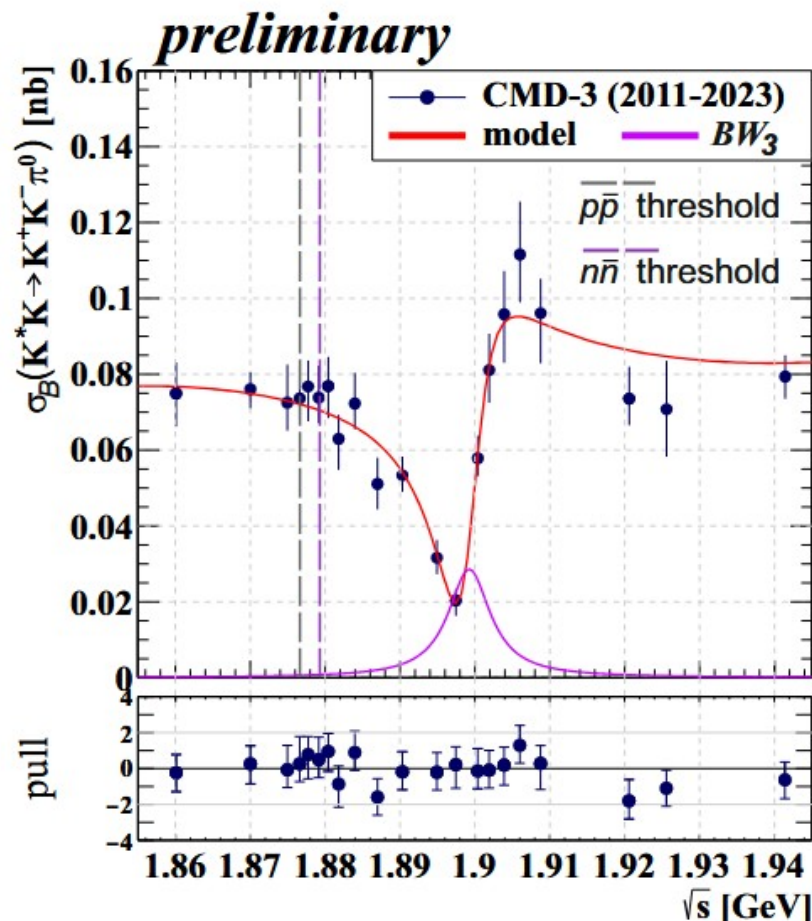
Preliminary fit: 16 parameters. The phase of the $\rho(1700)$ resonance is fixed by isospin symmetry. Separation of isoscalar and isovector amplitudes does not yet seem possible.

	mass [GeV]	width [GeV]
BW_1	1.752 ± 0.008	0.194 ± 0.01
BW_2	1.598 ± 0.005	0.325 ± 0.015
$\rho(1700)$	1.72 (fixed)	0.25 (fixed)
$\omega(1650)$	1.67 (fixed)	0.315 (fixed)
$\phi(2170)$	2.175 (fixed)	0.061 (fixed)
BW_3	1.8992 ± 0.0013	0.0069 ± 0.0024



(12/13) $\rho(1900)$?

Parameters of the introduced resonance:
 $m = 1.8992 \pm 0.0013$ GeV and $\Gamma = 6.9 \pm 2.4$ MeV.



$\rho(1900)$

$$J^{PC} = 1^+(1^{--})$$

OMITTED FROM SUMMARY TABLE

See the review on "Spectroscopy of Light Meson Resonances."

$\rho(1900)$ MASS

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •				
1880 ± 10		¹ ABLIKIM	22L BES3	$2.0-3.08 e^+e^- \rightarrow K^+K^-\pi^0$
$1909 \pm 17 \pm 25$	54	² AUBERT	08S BABR	$10.6 e^+e^- \rightarrow \phi\pi^0\gamma$
1880 ± 30		AUBERT	06D BABR	$10.6 e^+e^- \rightarrow 3\pi^+3\pi^-\gamma$
1860 ± 20		AUBERT	06D BABR	$10.6 e^+e^- \rightarrow 2(\pi^+\pi^-\pi^0)\gamma$
1910 ± 10		^{3,4} FRABETTI	04 E687	$\gamma p \rightarrow 3\pi^+3\pi^-\rho$
1870 ± 10		ANTONELLI	96 SPEC	$e^+e^- \rightarrow \text{hadrons}$

¹ From a partial wave amplitude analysis at $\sqrt{s} = 2.125$ GeV which includes all the possible intermediate states that match J^{PC} conservation in the subsequent two-body decay. The intermediate states are parameterized with the relativistic Breit-Wigner functions. Statistical error only.

² From the fit with two resonances.

³ From a fit with two resonances with the JACOB 72 continuum.

⁴ Supersedes FRABETTI 01.

$\rho(1900)$ WIDTH

VALUE (MeV)	EVTS	DOCUMENT ID	TECN	COMMENT
• • • We do not use the following data for averages, fits, limits, etc. • • •				
69 ± 15		¹ ABLIKIM	22L BES3	$2.0-3.08 e^+e^- \rightarrow K^+K^-\pi^0$
$48 \pm 17 \pm 2$	54	² AUBERT	08S BABR	$10.6 e^+e^- \rightarrow \phi\pi^0\gamma$
130 ± 30		AUBERT	06D BABR	$10.6 e^+e^- \rightarrow 3\pi^+3\pi^-\gamma$
160 ± 20		AUBERT	06D BABR	$10.6 e^+e^- \rightarrow 2(\pi^+\pi^-\pi^0)\gamma$
37 ± 13		^{3,4} FRABETTI	04 E687	$\gamma p \rightarrow 3\pi^+3\pi^-\rho$
10 ± 5		ANTONELLI	96 SPEC	$e^+e^- \rightarrow \text{hadrons}$

¹ From a partial wave amplitude analysis at $\sqrt{s} = 2.125$ GeV which includes all the possible intermediate states that match J^{PC} conservation in the subsequent two-body decay. The intermediate states are parameterized with the relativistic Breit-Wigner functions. Statistical error only.

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