Phenomenology of near-threshold states in the spectrum of heavy quarks

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in collaboration with

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Introduction

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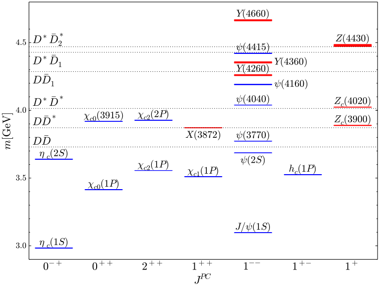
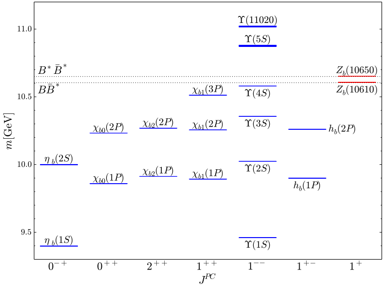




Many hadronic states are found in spectrum of heavy quarks which

- do not fit into the quark model scheme
- reside near S-wave open-flavour thresholds
- have large decay branchings to nearby channels

These are strong candidates to hadronic molecules

(to be distinguished from tetraquarks, Esposito et al (2014))

The ultimate goal

- To formulate field theoretical approach to near-threshold states which respects
  - multichannel dynamics
  - unitarity
  - analyticity
- To build simple but phenomenologically adequate formalism for combined data analysis for exotic states
- To build chiral extrapolations for exotic near-threshold states
- To extract parameters of near-threshold resonances directly from data and lattice calculations
- To predict new exotic states

Why not just Breit-Wigners?

Note:

BW implies substitution loop operator $\to$ constant width

Then:

- No threshold phenomena in BW
  (Im part does not change across threshold)
- Notions "mass" and "width" are misleading near threshold(s)
  (e.g. for cusp $M_{\rm peak} = M_{\rm threshold}$ and $\Gamma_{\rm visible} < \sum \Gamma_{\rm partial}$)
- BW has problems with analyticity
  (Only one pole of two symmetric poles is picked up. This works fine near the resonance but both poles are important near threshold)
- Naive sum of BW's violates unitarity
  ($\text{Im}(BW) \propto |BW|^2$ but $\text{Im}(BW_1 + BW_2) \not\propto |BW_1 + BW_2|^2$)

Conclusion:

BW's should never be used for near-threshold states

Heavy-quark spin symmetry

- Exotic XYZ states contain heavy quarks (HQ)
- In the limit $m_Q \to \infty$ $(m_Q \gg \Lambda_{\rm QCD})$ spin of HQ decouples

  $\Longrightarrow$ Heavy Quark Spin Symmetry (HQSS)
- For realistic m_Q's HQSS is approximate but rather accurate symmetry of QCD
- HQSS is a tool to study properties of states with different HQ spin orientation

  $\Longrightarrow$ Spin partners
- Predictions of HQSS depend crucially on the nature of states under study

  (Cleven et al (2015))
- Disclaimer: In this talk, only molecular scenario is discussed
- Quarkonium component of the w.f. (if exists) may impact the predictions

  (Cincioglu et al. (2016))

Coupled-channel approach

Coupled-channel approach to near-threshold states

- Coupled-channel problem is formulated in the most general form

$$|\Psi\rangle = \sum_{a=1}^{N_\mathrm{p}} \sqrt{Z_a}|(\bar{Q}Q)\rangle_a + \sum_{\alpha=1}^{N_\mathrm{e}} \psi_\alpha(p)|(\bar{Q}q)(\bar{q}Q)\rangle_\alpha + \sum_{i=1}^{N_\mathrm{in}} \psi_i(k)|(\bar{Q}Q)(\bar{q}q)\rangle_i$$

  - N_p = # bare poles (quark states)
  - N_e = # elastic (open-flavour) channels
  - N_in = # inelastic (hidden-flavour) channels

- Lippmann-Schwinger equations guarantee a correct account for
  - Unitarity (all channels are iterated to all orders)
  - Threshold phenomena (width $\to$ loop operator)
  - Analyticity (both Re(loop) and Im(loop) are kept)

Coupled-channel problem for $X(3872)$ and $Z_b^{(\prime)}$

	$Z_b(10610)$, $Z_b(10650)$	$X(3872)$
Production reaction	$\Upsilon(5S) \to \pi Z_b^{(\prime)}$	$B \to KX$
Quark component	—	χ'_{c1}
Elastic channels analysed	$B\bar{B}^*$, $B^*\bar{B}^*$	$D\bar{D}^*$
Inelastic channels analysed	$\pi h_b(1P)$, $\pi h_b(2P)$	$\pi\pi J/\psi$
Inelastic branchings used	$\pi\Upsilon(1S)$, $\pi\Upsilon(2S)$, $\pi\Upsilon(3S)$	$\pi\pi\pi J/\psi$, $\gamma J/\psi$, $\gamma\psi'$
Not measured inelastic modes	—	Hadronic modes χ'_{c1} (additional inelasticity)
Summary	$N_{\rm p}=0,\ N_{\rm e}=2,\ N_{\rm in}=5$	$N_{\rm p}=1,\ N_{\rm e}=1,\ N_{\rm in}=4$

Practical parametrisation

Coupled channels: Problems and solutions

Problems:

- Typically, $N_\mathrm{p} = 0..2$, $N_\mathrm{e} = 1..2$ however $N_\mathrm{in} \gg 1$
- Extra inelastic channels entail reformulation of entire problem
- LSE cannot be solved analytically in general terms

Simplifications:

- Neglect direct interaction between inelastic channels (for example, $\rho(\bar{Q}Q) \leftrightarrow \omega(\bar{Q}Q)$ or $\pi(\bar{Q}Q) \leftrightarrow \pi(\bar{Q}Q)$)
- Assume elastic-to-inelastic form factors in a separable form

Outcome:

- All channels involved are completely disentangled
- LSE are solved analytically; solution $\rightarrow$ parametrisation
- Inelastic channels enter additively (e.g. $\sum_{i=1}^{N_\mathrm{in}}$ )
- The problem reduces to matrices $N_\mathrm{e} \times N_\mathrm{e}$ and $N_\mathrm{p} \times N_\mathrm{p}$

Practical parametrisation

- Direct interaction elastic t matrix [2 parameters—see below]
- Couplings

Vertex	Transition
$v_{\alpha a}$	elastic S-wave channels $\Leftrightarrow$ bare poles
$v_{ai}(\boldsymbol{k}) = \lambda_{ai}\lvert\boldsymbol{k}\rvert^{l_i}$	inelastic l_i-wave channels $\Leftrightarrow$ bare poles
$v_{i\alpha}(\boldsymbol{k}) = g_{i\alpha}\lvert\boldsymbol{k}\rvert^{l_i}$	S-wave elastic $\Leftrightarrow$ l_i-wave inelastic channels

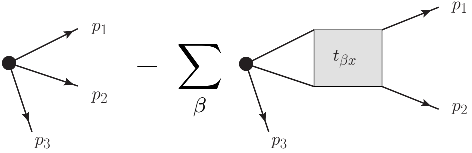
$$\left[\begin{array}{ll} g_{[\pi\Upsilon(nS)][B^{(*)}\bar{B}^*]}\ (n=1,2,3) & \text{6 parameters} \\ g_{[\pi h_b(mP)][B^{(*)}\bar{B}^*]}\ (m=1,2) & \text{4 parameters} \end{array}\right]$$

- Ratios of production sources ξ_α $\left[\xi = \frac{g_{[\pi\Upsilon(5S)][B^*\bar{B}^*]}}{g_{[\pi\Upsilon(5S)][B\bar{B}^*]}}\right]$
- Norm in each distribution [7 channels = 7 norms]

Note!

- All parameters are real, imaginary parts come from loops
- If additional inelasticity is needed then data set is incomplete

Differential rates



$$\frac{d\text{Br}[B\bar{B}^*]}{dM} = \mathcal{N}_{B\bar{B}^*}\left|t_{11} + \xi t_{21}\right|^2 p_\pi k_{B\bar{B}^*}$$

$$\frac{d\text{Br}[B^*\bar{B}^*]}{dM} = \mathcal{N}_{B^*\bar{B}^*}\left|t_{12} + \xi t_{22}\right|^2 p_\pi k_{B^*\bar{B}^*}$$

$$\frac{d\text{Br}[\pi\Upsilon(nS)]}{dM} = \mathcal{N}_{\pi\Upsilon(nS)}\left|(t_{11}+\xi t_{21})+\frac{g_{[B^*\bar{B}^*][\pi\Upsilon(nS)]}}{g_{[B\bar{B}^*][\pi\Upsilon(nS)]}}(t_{12}+\xi t_{22})\right|^2 p_\pi k_{\pi\Upsilon(nS)}$$

$$\frac{d\text{Br}[\pi h_b(mP)]}{dM} = \mathcal{N}_{\pi h_b(mP)}\left|(t_{11}+\xi t_{21})+\frac{g_{[B^*\bar{B}^*][\pi h_b(mP)]}}{g_{[B\bar{B}^*][\pi h_b(mP)]}}(t_{12}+\xi t_{22})\right|^2 p_\pi k^3_{\pi h_b(mP)}$$

Constraints from Heavy Quark Spin Symmetry

- Spin w.f.'s of $B^{(*)}\bar{B}^*$ pairs with quantum numbers 1^{+-}

$$|B\bar{B}^*\rangle = 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q} + 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q}$$
$$|B^*\bar{B}^*\rangle = 0^-_{\bar{b}b} \otimes 1^-_{\bar{q}q} - 1^-_{\bar{b}b} \otimes 0^-_{\bar{q}q}$$

$$\Longrightarrow \frac{g_{[\pi h_b(mP)][B^*\bar{B}^*]}}{g_{[\pi h_b(mP)][B\bar{B}^*]}} = -\frac{g_{[\pi\Upsilon(nS)][B^*\bar{B}^*]}}{g_{[\pi\Upsilon(nS)][B\bar{B}^*]}} = 1$$

A.E. Bondar et al. PRD 84 (2011) 054010

- Direct interaction elastic potential

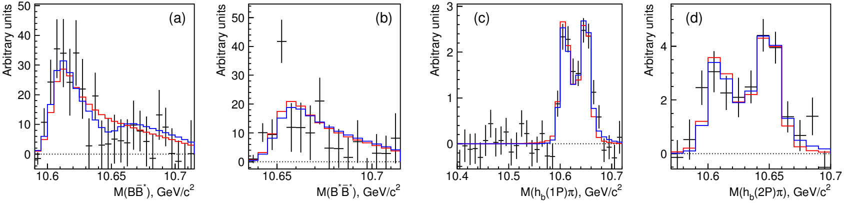
$$V(1^{+-}) = \begin{pmatrix} V_{B\bar{B}^*\to B\bar{B}^*} & V_{B\bar{B}^*\to B^*\bar{B}^*} \\ V_{B^*\bar{B}^*\to B\bar{B}^*} & V_{B^*\bar{B}^*\to B^*\bar{B}^*} \end{pmatrix}$$

$$\propto \begin{pmatrix} \gamma_s^{-1} + \gamma_t^{-1} & \gamma_s^{-1} - \gamma_t^{-1} \\ \gamma_s^{-1} - \gamma_t^{-1} & \gamma_s^{-1} + \gamma_t^{-1} \end{pmatrix}$$

Fits for the data

Fit	γ_s,MeV	γ_t,MeV	ξ	$\frac{g_{[\pi h_b(1P)][B^*\bar{B}^*]}}{g_{[\pi h_b(1P)][B\bar{B}^*]}}$	$\frac{g_{[\pi h_b(2P)][B^*\bar{B}^*]}}{g_{[\pi h_b(2P)][B\bar{B}^*]}}$	C.L.
A	35^{+38}_{-56}	-228^{+68}_{-61}	$-0.83^{+0.08}_{-0.07}$	$1.73^{+0.68}_{-0.42}$	$1.72^{+0.70}_{-0.43}$	55%
B	-86^{+32}_{-36}	-93^{+35}_{-39}	-1^*	1^*	1^*	47%

* Constrained from HQSS (7 norms + 7 parameters for shapes)



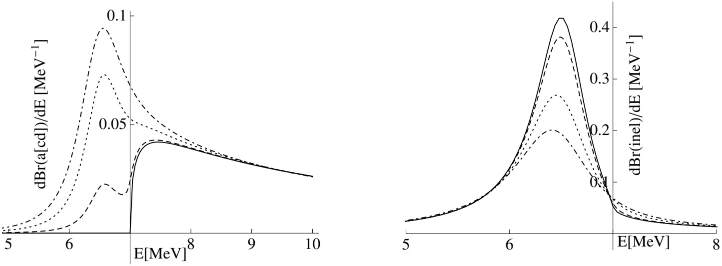
Data:

A. Garmash et al. [Belle Collab.], PRL 116 (2016) 212001 [arXiv:1512.07419]

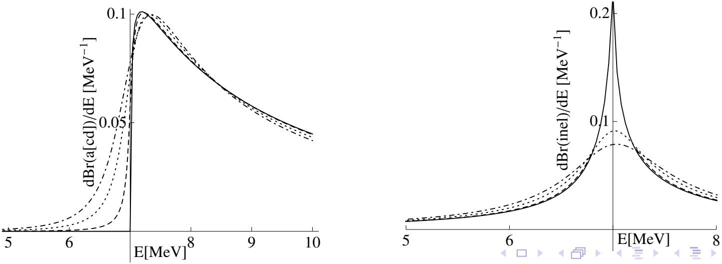
A. Bondar et al. [Belle Collab.], PRL 108 (2012) 122001 [arXiv:1110.2251]

Comment on line shapes: bound vs virtual state

Bound state



Virtual state



Perspectives and obstacles

- Additional production and decay mechanisms
- Final-state interaction
- Two-dimensional distributions to analyse Dalitz plot directly
- Additional interactions (one-pion exchange!)
- We need more accurate data!
- Synchronised notations and definitions between theory and experiment (partial BF's!)

Spin partners

Parameters and Input

- Short-range elastic interactions $\Longrightarrow$ Low-Energy Constants
- Transition potential between channels $\Longrightarrow$ Coupling constants
- Overall normalisation constants
- Bare poles (not necessary for Z_b's)

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HQ limit $\Longrightarrow$ Reduced # of parameters

- Ways to proceed
  - **Proper way:** combined coupled-channel fit for all measured channels
  - **Simplified way:** LEC's fixed to binding energies of known resonances

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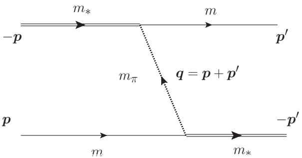
Binding energies of $X(3872)$ and $Z_b(10610)/Z_b(10650)$ will be used as input

One-pion exchange

- Pionic Lagrangian

$$\mathcal{L} = \frac{g_Q}{2f_\pi}\left(\boldsymbol{V}^\dagger \cdot \boldsymbol{\nabla}\pi^a\tau^a P + P^\dagger\tau^a\boldsymbol{\nabla}\pi^a \cdot \boldsymbol{V} + i[\boldsymbol{V}^\dagger \times \boldsymbol{V}] \cdot \boldsymbol{\nabla}\pi^a\tau^a\right)$$

- OPE potential



$$V^{ij}_{P\bar{V}\to\bar{P}V}(\boldsymbol{p},\boldsymbol{p}') = -\frac{g_Q^2}{(4\pi f_\pi)^2}(\boldsymbol{\tau}\cdot\boldsymbol{\tau}^c)\frac{q_i q_j}{D_3(\boldsymbol{p},\boldsymbol{p}')}$$

$$E_\pi = \sqrt{\boldsymbol{q}^2 + m_\pi^2}$$

$$D_3(\boldsymbol{p},\boldsymbol{p}') = 2E_\pi\left[\left(m + \frac{\boldsymbol{p}^2}{2m} + m + \frac{\boldsymbol{p}'^2}{2m} + E_\pi\right) - (m_* + m + E)\right]$$

- When $m_* > m + m_\pi \implies$ Three-body cut
- When recoil terms neglected $\implies$ Static OPE
- When $q_i q_j \to \frac{1}{3}\boldsymbol{q}^2\delta_{ij} \implies$ Central (S-wave) OPE

Spin partners: OPE included on top of LEC's

- P-wave $V \to P(V)\pi$ vertices $\Longrightarrow$ Extended basis

$$\begin{aligned}
0^{++}: &\quad \{P\bar{P}(^1S_0), V\bar{V}(^1S_0), V\bar{V}(^5D_0)\} \\
1^{+-}: &\quad \{P\bar{V}(^3S_1,-), P\bar{V}(^3D_1,-), V\bar{V}(^3S_1), V\bar{V}(^3D_1)\} \\
1^{++}: &\quad \{P\bar{V}(^3S_1,+), P\bar{V}(^3D_1,+), V\bar{V}(^5D_1)\} \\
2^{++}: &\quad \{P\bar{P}(^1D_2), P\bar{V}(^3D_2), V\bar{V}(^5S_2), V\bar{V}(^1D_2), V\bar{V}(^5D_2), V\bar{V}(^5G_2)\}
\end{aligned}$$

Important and cannot be ignored:

- Coupled-channel dynamics
- High momenta ($q \sim 500$ MeV)
- D waves (q^2/m_π^2 is large)
- Three-body dynamics (c-sector)

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1^{++}: &\quad \{P\bar{V}(^3S_1,+), P\bar{V}(^3D_1,+), V\bar{V}(^5D_1)\}\\
2^{++}: &\quad \{P\bar{P}(^1D_2), P\bar{V}(^3D_2), V\bar{V}(^5S_2), V\bar{V}(^1D_2), V\bar{V}(^5D_2), V\bar{V}(^5G_2)\}
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$$

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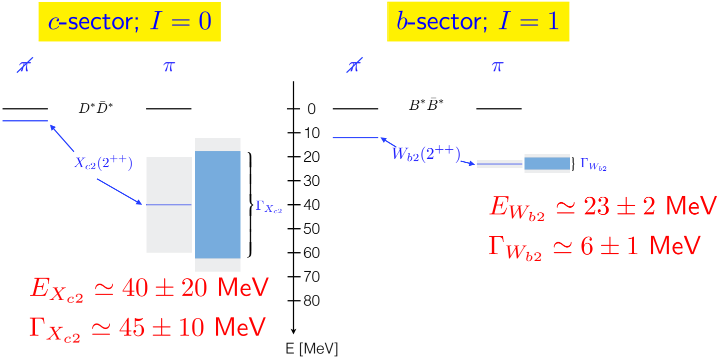
OPE couples 2^{++} channel to other channels $\Longrightarrow$ finite width

Spin partners: Results for the 2^{++} state

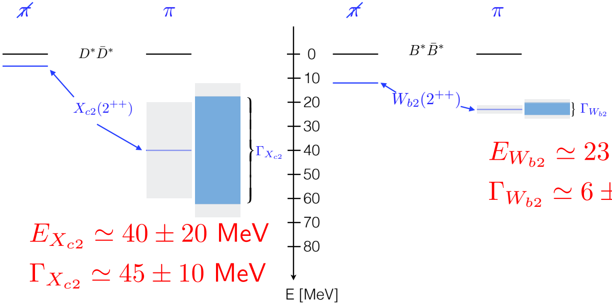


- 2 inputs in isovector b-sector $\Longrightarrow$ predictions for all spin partners
- 1 input in isosinglet c-sector $\Longrightarrow$ only 2^{++} partner can be predicted

Spin partners: Results for the 2^{++} state

c-sector; $I = 0$

b-sector; $I = 1$



$E_{X_{c2}} \simeq 40 \pm 20$ MeV

$\Gamma_{X_{c2}} \simeq 45 \pm 10$ MeV

$E_{W_{b2}} \simeq 23 \pm 2$ MeV

$\Gamma_{W_{b2}} \simeq 6 \pm 1$ MeV

- 2 inputs in isovector b-sector $\Longrightarrow$ predictions for all spin partners
- 1 input in isosinglet c-sector $\Longrightarrow$ only 2^{++} partner can be predicted

We need second input for $I = 0$ in c-sector — $X(3915)$?

$X(3915)$: Where are we now?

Experimental background (Belle & BaBar)

- Seen in $B \to KX \to K(\omega J/\psi)$ and $\gamma\gamma \to X \to \omega J/\psi$
- Not seen in $D\bar{D}$ mode

Quantum numbers

- Belle: 0^{++} or 2^{++}
- BaBar: 2^{++} ruled out by angular analysis in $\omega J/\psi$ (biased!)

Possible identification

- Not genuine $\chi_{c0}(2P)$ as
  - Does not fit expected properties of $\chi_{c0}(2P)$
  - There is good candidate $X^*(3860)$
- Exotic 0^{++} or 2^{++} state?

Molecule model

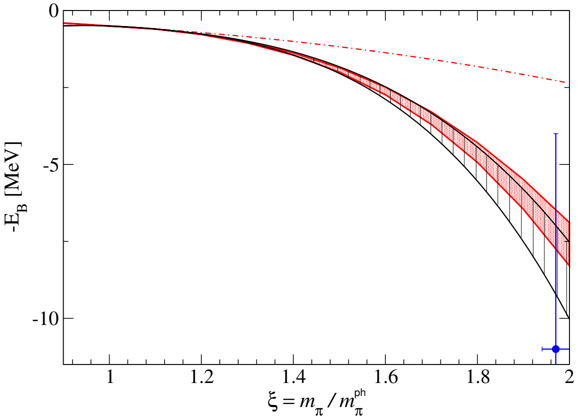
- Not 2^{++} $D^*\bar{D}^*$ molecule
- Might be 0^{++} $D^*\bar{D}^*$ molecule but E_B too large (~ 100 MeV)
- Good candidate for 0^{++} $D_s\bar{D}_s$ molecule $\Longrightarrow$ SU(3) extension

Perspectives and obstacles

- Relation between LEC's in different heavy-quark sectors
- Extension to light-flavour SU(3) group
- We need more input in channels with different heavy quarks (c and b), with different isospins, with strangeness!
- We need more data for decay modes of near-threshold states to fix parameters to observables
- We need more exclusive measurements to distinguish between genuine $\bar{Q}Q$ and exotic states

Chiral extrapolation

Chiral extrapolation for $X(3872)$ binding energy



- Black band: nonrelativistic approach
- Red band: relativistic approach
- Red dashed-dotted line: pionless theory
- Blue dot with error bar: lattice result

Conclusions

Conclusions

- The proposed systematic approach to hadronic molecules respects all relevant symmetries (chiral symmetry, HQSS, unitarity, analyticity) and allows to
  - build a practical parametrisation for line shapes
  - extract parameters of resonances directly from data
  - build chiral extrapolations
  - investigate various molecular candidates in c- and b-sectors
- HQSS breaking and nonperturbative pions have significant impact on near-threshold states
- Further theoretical work is in progress...
- We strongly need more data on exotic near-threshold states!