Phenomenology of near-threshold states in the spectrum of heavy quarks

A.V. Nefediev

(ITEP, Moscow)

in collaboration with

Introduction
Many hadronic states are found in spectrum of heavy quarks which

- do not fit into the quark model scheme
- reside near $S$-wave open-flavour thresholds
- have large decay branchings to nearby channels

These are strong candidates to **hadronic molecules**

(to be distinguished from tetraquarks, Esposito et al (2014))
The ultimate goal

- To formulate field theoretical approach to near-threshold states which respects
  - multichannel dynamics
  - unitarity
  - analyticity

- To build **simple** but **phenomenologically adequate** formalism for **combined** data analysis for exotic states

- To build **chiral extrapolations** for exotic near-threshold states

- To extract **parameters** of near-threshold resonances directly from **data** and lattice **calculations**

- To **predict** new exotic states
Why not just Breit-Wigners?

**Note:**

BW implies substitution loop operator $\rightarrow$ constant width

**Then:**

- No threshold phenomena in BW
  (Im part does not change across threshold)
- Notions “mass” and “width” are misleading near threshold(s)
  (e.g. for cusp $M_{\text{peak}} = M_{\text{threshold}}$ and $\Gamma_{\text{visible}} < \sum \Gamma_{\text{partial}}$)
- BW has problems with analyticity
  (Only one pole of two symmetric poles is picked up. This works fine near the resonance but both poles are important near threshold)
- Naive sum of BW’s violates unitarity
  ($\text{Im}(BW) \propto |BW|^2$ but $\text{Im}(BW_1 + BW_2) \not\propto |BW_1 + BW_2|^2$)

**Conclusion:**

BW’s should never be used for near-threshold states
Heavy-quark spin symmetry

- Exotic $XYZ$ states contain heavy quarks (HQ)

- In the limit $m_Q \to \infty$ ($m_Q \gg \Lambda_{\text{QCD}}$) spin of HQ decouples
  \[ \Rightarrow \text{Heavy Quark Spin Symmetry (HQSS)} \]

- For realistic $m_Q$'s HQSS is approximate but rather accurate symmetry of QCD

- HQSS is a tool to study properties of states with different HQ spin orientation
  \[ \Rightarrow \text{Spin partners} \]

- Predictions of HQSS depend crucially on the nature of states under study
  \[ \text{(Cleven et al (2015))} \]

- Disclaimer: In this talk, only molecular scenario is discussed

- Quarkonium component of the w.f. (if exists) may impact the predictions
  \[ \text{(Cincioglu et al. (2016))} \]
Coupled-channel approach
**Coupled-channel approach to near-threshold states**

- **Coupled-channel problem is formulated in the most general form**

\[ |\Psi\rangle = \sum_{a=1}^{N_p} \sqrt{Z_a} |(\bar{Q}Q)\rangle_a + \sum_{\alpha=1}^{N_e} \psi_\alpha(p) |(\bar{Q}q)(\bar{q}Q)\rangle_\alpha + \sum_{i=1}^{N_{in}} \psi_i(k) |(\bar{Q}Q)(\bar{q}q)\rangle_i \]

- \( N_p = \# \) bare poles (quark states)
- \( N_e = \# \) elastic (open-flavour) channels
- \( N_{in} = \# \) inelastic (hidden-flavour) channels

- Lippmann-Schwinger equations guarantee a correct account for
  - **Unitarity** (all channels are iterated to all orders)
  - **Threshold phenomena** (width \(\to\) loop operator)
  - **Analyticity** (both \(\text{Re}(\text{loop})\) and \(\text{Im}(\text{loop})\) are kept)
## Coupled-channel problem for $X(3872)$ and $Z_b^{(1)}$

<table>
<thead>
<tr>
<th>Production reaction</th>
<th>$Z_b(10610)$, $Z_b(10650)$</th>
<th>$X(3872)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quark component</td>
<td>$\Upsilon(5S) \to \pi Z_b^{(1)}$</td>
<td>$B \to K X$</td>
</tr>
<tr>
<td>Elastic channels</td>
<td>$B \bar{B}^<em>$, $B^</em> \bar{B}^*$</td>
<td>$\chi_{c1}'$</td>
</tr>
<tr>
<td>analyised</td>
<td></td>
<td>$D \bar{D}^*$</td>
</tr>
<tr>
<td>Inelastic channels</td>
<td>$\pi h_b(1P), \pi h_b(2P)$</td>
<td>$\pi \pi J/\psi$</td>
</tr>
<tr>
<td>analyised</td>
<td>$\pi \Upsilon(1S), \pi \Upsilon(2S), \pi \Upsilon(3S)$</td>
<td>$\pi \pi \pi J/\psi, \gamma J/\psi, \gamma \psi'$</td>
</tr>
<tr>
<td>Inelastic branchings</td>
<td></td>
<td>Hadronic modes $\chi_{c1}'$ (additional inelasticity)</td>
</tr>
<tr>
<td>Not measured</td>
<td></td>
<td></td>
</tr>
<tr>
<td>inelastic modes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Summary</td>
<td>$N_p = 0, N_e = 2, N_{in} = 5$</td>
<td>$N_p = 1, N_e = 1, N_{in} = 4$</td>
</tr>
</tbody>
</table>
Practical parametrisation
Coupled channels: Problems and solutions

Problems:

- Typically, $N_p = 0..2$, $N_e = 1..2$ however $N_{in} \gg 1$
- Extra inelastic channels entail reformulation of entire problem
- LSE cannot be solved analytically in general terms

Simplifications:

- Neglect direct interaction between inelastic channels (for example, $\rho(\bar{Q}Q) \leftrightarrow \omega(\bar{Q}Q)$ or $\pi(\bar{Q}Q) \leftrightarrow \pi(\bar{Q}Q)$)
- Assume elastic-to-inelastic form factors in a separable form

Outcome:

- All channels involved are completely disentangled
- LSE are solved analytically; solution $\rightarrow$ parametrisation
- Inelastic channels enter additively (e.g. $\sum_{i=1}^{N_{in}} \bigotimes$)
- The problem reduces to matrices $N_e \times N_e$ and $N_p \times N_p$
Practical parametrisation

- **Direct interaction elastic** $t$ **matrix** [2 parameters—see below]
- **Couplings**

<table>
<thead>
<tr>
<th>Vertex</th>
<th>Transition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_{\alpha a}$</td>
<td>elastic $S$-wave channels $\Leftrightarrow$ bare poles</td>
</tr>
<tr>
<td>$v_{ai}(k) = \lambda_{ai}</td>
<td>k</td>
</tr>
<tr>
<td>$v_{i\alpha}(k) = g_{i\alpha}</td>
<td>k</td>
</tr>
</tbody>
</table>

$$
\begin{bmatrix}
    g[\pi \Upsilon(nS)][B^{(*)}\bar{B}^{(*)}] (n = 1, 2, 3) \\
    g[\pi h_b(mP)][B^{(*)}\bar{B}^{(*)}] (m = 1, 2)
\end{bmatrix}
$$

- **Ratios of production sources** $\xi_{\alpha}$
  $$\xi = \frac{g[\pi \Upsilon(5S)][B^{*}\bar{B}^{*}]}{g[\pi \Upsilon(5S)][B\bar{B}^{*}]}$$

- **Norm in each distribution** [7 channels $= 7$ norms]

**Note!**
- All parameters are **real**, imaginary parts come from loops
- If additional inelasticity is needed then data set is incomplete
**Differential rates**

\[
\frac{d\text{Br}[B\bar{B}^*]}{dM} = \mathcal{N}_{B\bar{B}^*} \left| t_{11} + \xi t_{21} \right|^2 p_\pi k_{B\bar{B}^*}
\]

\[
\frac{d\text{Br}[B^*\bar{B}^*]}{dM} = \mathcal{N}_{B^*\bar{B}^*} \left| t_{12} + \xi t_{22} \right|^2 p_\pi k_{B^*\bar{B}^*}
\]

\[
\frac{d\text{Br}[\pi\Upsilon(nS)]}{dM} = \mathcal{N}_{\pi\Upsilon(nS)} \left( t_{11} + \xi t_{21} \right) + \frac{g_{[B^*\bar{B}^*][\pi\Upsilon(nS)]}}{g_{[B\bar{B}^*][\pi\Upsilon(nS)]}} \left( t_{12} + \xi t_{22} \right)^2 p_\pi k_{\pi\Upsilon(nS)}
\]

\[
\frac{d\text{Br}[\pi h_b(mP)]}{dM} = \mathcal{N}_{\pi h_b(mP)} \left( t_{11} + \xi t_{21} \right) + \frac{g_{[B^*\bar{B}^*][\pi h_b(mP)]}}{g_{[B\bar{B}^*][\pi h_b(mP)]}} \left( t_{12} + \xi t_{22} \right)^2 p_\pi k_{\pi h_b(mP)}
\]
Constraints from Heavy Quark Spin Symmetry

- Spin w.f.'s of $B^{(*)} \bar{B}^*$ pairs with quantum numbers $1^{+-}$

$$
|B\bar{B}^*\rangle = 0_{bb}^- \otimes 1_{qq}^- + 1_{bb}^- \otimes 0_{qq}^-
$$

$$
|B^*\bar{B}^*\rangle = 0_{bb}^- \otimes 1_{qq}^- - 1_{bb}^- \otimes 0_{qq}^-
$$

$$
\Rightarrow \frac{g[\pi h_b(mP)][B^*\bar{B}^*]}{g[\pi h_b(mP)][B\bar{B}^*]} = -\frac{g[\pi \Upsilon(nS)][B^*\bar{B}^*]}{g[\pi \Upsilon(nS)][B\bar{B}^*]} = 1
$$

- Direct interaction elastic potential

$$
V(1^{+-}) = \begin{pmatrix}
V_{B\bar{B}^*\rightarrow B\bar{B}^*} & V_{B\bar{B}^*\rightarrow B^*\bar{B}^*} \\
V_{B^*\bar{B}^*\rightarrow B\bar{B}^*} & V_{B^*\bar{B}^*\rightarrow B^*\bar{B}^*}
\end{pmatrix}
$$

$$
\propto \begin{pmatrix}
\gamma_s^{-1} + \gamma_t^{-1} & \gamma_s^{-1} - \gamma_t^{-1} \\
\gamma_s^{-1} - \gamma_t^{-1} & \gamma_s^{-1} + \gamma_t^{-1}
\end{pmatrix}
$$

A.E. Bondar et al. PRD 84 (2011) 054010
### Fits for the data

<table>
<thead>
<tr>
<th>Fit</th>
<th>$\gamma_s$, MeV</th>
<th>$\gamma_t$, MeV</th>
<th>$\xi$</th>
<th>$\frac{g[\pi h_b(1P)][B^<em>\bar{B}^</em>]}{g[\pi h_b(1P)][B\bar{B}^*]}$</th>
<th>$\frac{g[\pi h_b(2P)][B^<em>\bar{B}^</em>]}{g[\pi h_b(2P)][B\bar{B}^*]}$</th>
<th>C.L.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$35^{+38}_{-56}$</td>
<td>$-228^{+68}_{-61}$</td>
<td>$-0.83^{+0.08}_{-0.07}$</td>
<td>$1.73^{+0.68}_{-0.42}$</td>
<td>$1.72^{+0.70}_{-0.43}$</td>
<td>55%</td>
</tr>
<tr>
<td>B</td>
<td>$-86^{+32}_{-36}$</td>
<td>$-93^{+35}_{-39}$</td>
<td>$-1^*$</td>
<td>$1^*$</td>
<td>$1^*$</td>
<td>47%</td>
</tr>
</tbody>
</table>

* Constrained from HQSS (7 norms + 7 parameters for shapes)

---

**Data:**

Comment on line shapes: bound vs virtual state

**Bound state**

**Virtual state**
Perspectives and obstacles

- Additional production and decay mechanisms
- Final-state interaction
- Two-dimensional distributions to analyse Dalitz plot directly
- Additional interactions (one-pion exchange!)
- We need more accurate data!
- Synchronised notations and definitions between theory and experiment (partial BF’s!)
Spin partners
Parameters and Input

- Short-range elastic interactions $\rightarrow$ Low-Energy Constants
- Transition potential between channels $\rightarrow$ Coupling constants
- Overall normalisation constants
- Bare poles (not necessary for $Z_b$’s)
Parameters and Input

- Short-range elastic interactions \( \Rightarrow \) Low-Energy Constants
- Transition potential between channels \( \Rightarrow \) Coupling constants
- Overall normalisation constants
- Bare poles (not necessary for \( Z_b \)’s)

\[ \text{HQ limit} \quad \Rightarrow \quad \text{Reduced \# of parameters} \]

- Ways to proceed
  - **Proper way:** combined coupled-channel fit for all measured channels
  - **Simplified way:** LEC’s fixed to binding energies of known resonances
Parameters and Input

- Short-range elastic interactions $\implies$ Low-Energy Constants
- Transition potential between channels $\implies$ Coupling constants
- Overall normalisation constants
- Bare poles (not necessary for $Z_b$’s)
  
  HQ limit $\implies$ Reduced # of parameters

- Ways to proceed
  
  - **Proper way:** combined coupled-channel fit for all measured channels
  - **Simplified way:** LEC’s fixed to binding energies of known resonances

Binding energies of $X(3872)$ and $Z_b(10610)/Z_b(10650)$ will be used as input
One-pion exchange

- **Pionic Lagrangian**

\[
\mathcal{L} = \frac{g_Q}{2f_\pi} \left( V^\dagger \cdot \nabla \pi^a \tau^a P + P^\dagger \tau^a \nabla \pi^a \cdot V + i[V^\dagger \times V] \cdot \nabla \pi^a \tau^a \right)
\]

- **OPE potential**

\[
V_{PV \to \bar{P}V}(p, p') = -\frac{g_Q^2}{(4\pi f_\pi)^2} (\tau \cdot \tau^c) \frac{q_i q_j}{D_3(p, p')}
\]

\[
E_\pi = \sqrt{\mathbf{q}^2 + m_\pi^2}
\]

\[
D_3(p, p') = 2E_\pi \left[ \left( m + \frac{p^2}{2m} + m + \frac{p'^2}{2m} + E_\pi \right) - (m_* + m + E) \right]
\]

- When \( m_* > m + m_\pi \) \( \implies \) **Three-body cut**
- When recoil terms neglected \( \implies \) **Static OPE**
- When \( q_i q_j \to \frac{1}{3} q^2 \delta_{ij} \) \( \implies \) **Central (S-wave) OPE**
Spin partners: OPE included on top of LEC’s

- $P$-wave $V \rightarrow P(V)\pi$ vertices $\rightarrow$ Extended basis

0$^{++}$: $\{PP(1S_0), VV(1S_0), VV(5D_0)\}$
1$^{+-}$: $\{P\bar{V}(3S_1,-), P\bar{V}(3D_1,-), V\bar{V}(3S_1), V\bar{V}(3D_1)\}$
1$^{++}$: $\{P\bar{V}(3S_1,+), P\bar{V}(3D_1,+), V\bar{V}(5D_1)\}$
2$^{++}$: $\{PP(1D_2), P\bar{V}(3D_2), V\bar{V}(5S_2), V\bar{V}(1D_2), V\bar{V}(5D_2), V\bar{V}(5G_2)\}$

Important and cannot be ignored:

- Coupled-channel dynamics
- High momenta ($q \sim 500$ MeV)
- $D$ waves ($q^2/m_{\pi}^2$ is large)
- Three-body dynamics ($c$-sector)
Spin partners: OPE included on top of LEC’s

- $P$-wave $V \rightarrow P(V)\pi$ vertices $\Rightarrow$ Extended basis

0$^{++}$: $\{P\bar{P}(^1S_0), V\bar{V}(^1S_0), V\bar{V}(^5D_0)\}$

1$^{+-}$: $\{P\bar{V}(^3S_1, -), P\bar{V}(^3D_1, -), V\bar{V}(^3S_1), V\bar{V}(^3D_1)\}$

1$^{++}$: $\{P\bar{V}(^3S_1, +), P\bar{V}(^3D_1, +), V\bar{V}(^5D_1)\}$

2$^{++}$: $\{P\bar{P}(^1D_2), P\bar{V}(^3D_2), V\bar{V}(^5S_2), V\bar{V}(^1D_2), V\bar{V}(^5D_2), V\bar{V}(^5G_2)\}$

Important and cannot be ignored:

- Coupled-channel dynamics
- High momenta ($q \sim 500$ MeV)
- $D$ waves ($q^2/m^2_\pi$ is large)
- Three-body dynamics ($c$-sector)

OPE couples 2$^{++}$ channel to other channels $\Rightarrow$ finite width
Spin partners: Results for the $2^{++}$ state

$c$-sector; $I = 0$

$b$-sector; $I = 1$

$D^* \bar{D}^*$

$X_{c2}(2^{++})$

$E_{X_{c2}} \simeq 40 \pm 20$ MeV

$\Gamma_{X_{c2}} \simeq 45 \pm 10$ MeV

$B^* \bar{B}^*$

$W_{b2}(2^{++})$

$E_{W_{b2}} \simeq 23 \pm 2$ MeV

$\Gamma_{W_{b2}} \simeq 6 \pm 1$ MeV

- 2 inputs in isovector $b$-sector $\implies$ predictions for all spin partners
- 1 input in isosinglet $c$-sector $\implies$ only $2^{++}$ partner can be predicted
Spin partners: Results for the $2^{++}$ state

$c$-sector; $I = 0$

$b$-sector; $I = 1$

- $2^{++}$ partner can be predicted

2 inputs in isovector $b$-sector $\implies$ predictions for all spin partners

1 input in isosinglet $c$-sector $\implies$ only $2^{++}$ partner can be predicted

We need second input for $I = 0$ in $c$-sector — $X(3915)$?
**$X(3915)$: Where are we now?**

**Experimental background (Belle & BaBar)**
- **Seen** in $B \rightarrow KX \rightarrow K(\omega J/\psi)$ and $\gamma\gamma \rightarrow X \rightarrow \omega J/\psi$
- **Not seen** in $D\bar{D}$ mode

**Quantum numbers**
- Belle: $0^{++}$ or $2^{++}$
- BaBar: $2^{++}$ ruled out by angular analysis in $\omega J/\psi$ (biased!)

**Possible identification**
- **Not** genuine $\chi_{c0}(2P)$ as
  - Does not fit expected properties of $\chi_{c0}(2P)$
  - There is good candidate $X^*(3860)$
- Exotic $0^{++}$ or $2^{++}$ state?

**Molecule model**
- **Not** $2^{++}$ $D^*\bar{D}^*$ molecule
- **Might** be $0^{++}$ $D^*\bar{D}^*$ molecule but $E_B$ too large ($\sim 100$ MeV)
- **Good** candidate for $0^{++}$ $D_s\bar{D}_s$ molecule $\Rightarrow$ SU(3) extension
Perspectives and obstacles

- Relation between LEC’s in different heavy-quark sectors
- Extension to light-flavour SU(3) group
- We need more input in channels with different heavy quarks (c and b), with different isospins, with strangeness!
- We need more data for decay modes of near-threshold states to fix parameters to observables
- We need more exclusive measurements to distinguish between genuine $\bar{Q}Q$ and exotic states
Chiral extrapolation
Chiral extrapolation for $X(3872)$ binding energy

- Black band: nonrelativistic approach
- Red band: relativistic approach
- Red dashed-dotted line: pionless theory
- Blue dot with error bar: lattice result
Conclusions
Conclusions

- The proposed systematic approach to hadronic molecules respects all relevant symmetries (chiral symmetry, HQSS, unitarity, analyticity) and allows to
  - build a **practical parametrisation** for line shapes
  - extract **parameters** of resonances directly **from data**
  - build chiral extrapolations
  - investigate various **molecular candidates** in $c$- and $b$-sectors

- HQSS breaking and nonperturbative pions have significant impact on near-threshold states

- Further **theoretical** work is **in progress**...

- We strongly need more **data** on exotic near-threshold states!