Precision studies of leptonic τ decays

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3. Study of $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$ decay
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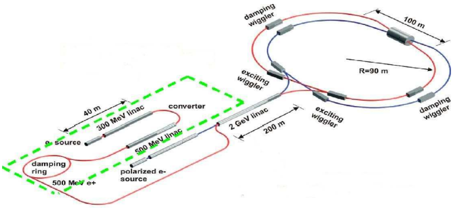
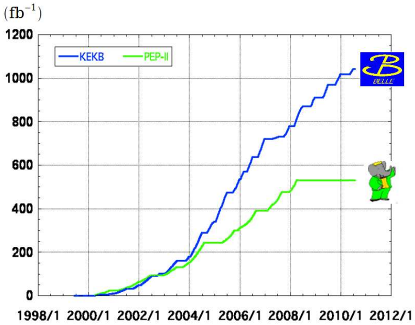
5. Summary



Introduction

- The world largest statistics of τ leptons collected by e^+e^- B factories (Belle and *BABAR*) opens new era in the precision tests of the Standard Model (SM).
- Basic tau properties, like: lifetime, mass, couplings, electric dipole moment, anomalous magnetic dipole moment, etc. should be measured experimentally as precisely as possible in order to test SM and search for the effects of New Physics.
- In the SM τ decays due to the charged weak interaction described by the exchange of $W^\pm$ with a pure vector coupling to only left-handed fermions. There are two main classes of tau decays:
  - Decays with leptons, like: $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$, $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau \gamma$, $\tau^- \to \ell^- \ell'^+ \ell'^- \bar{\nu}_\ell \nu_\tau$; $\ell, \ell' = e, \mu$. They provide very clean laboratory to probe electroweak couplings, which is complementary/competitive to precision studies with muon (in experiments with muon beam). Plenty of New Physics models can be tested/constrained in the precision studies of the dynamics of decays with leptons.
  - Hadronic decays of τ offer unique tools for the precision study of low energy QCD.

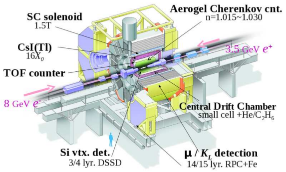
Integrated luminosity of B factories



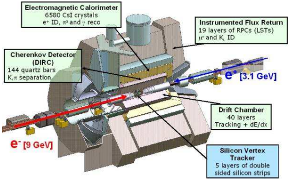
> 1 ab^{-1}
On resonance:
$\Upsilon(5S)$: 121 fb^{-1}
$\Upsilon(4S)$: 711 fb^{-1}
$\Upsilon(3S)$: 3 fb^{-1}
$\Upsilon(2S)$: 25 fb^{-1}
$\Upsilon(1S)$: 6 fb^{-1}
Off reson./scan:
~ 100 fb^{-1}

~ 550 fb^{-1}
On resonance:
$\Upsilon(4S)$: 433 fb^{-1}
$\Upsilon(3S)$: 30 fb^{-1}
$\Upsilon(2S)$: 14 fb^{-1}
Off resonance:
~ 54 fb^{-1}

Belle Detector



BABAR detector

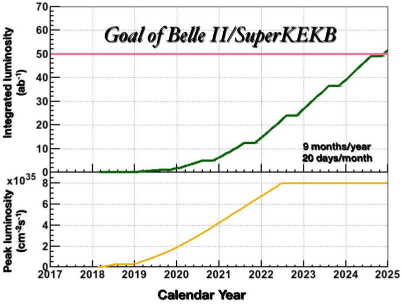


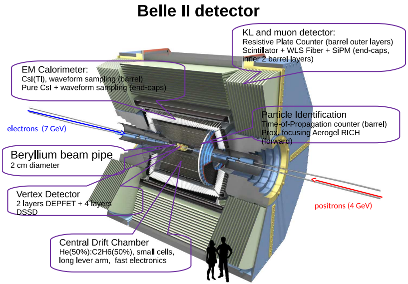
Integrated luminosity is 1.55 ab^{-1}

$$\sigma(b\bar{b}) = 1.05 \text{ nb} \quad N_{b\bar{b}} = 1.2 \times 10^9$$
$$\sigma(c\bar{c}) = 1.30 \text{ nb} \quad N_{c\bar{c}} = 2.0 \times 10^9$$
$$\boldsymbol{\sigma(\tau\tau) = 0.92 \text{ nb} \quad N_{\tau\tau} = 1.4 \times 10^9}$$

B factories are also charm and τ factories !

Introduction: Belle II

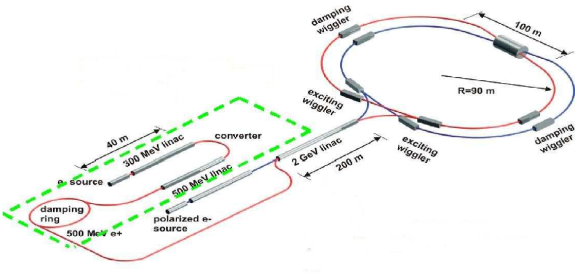


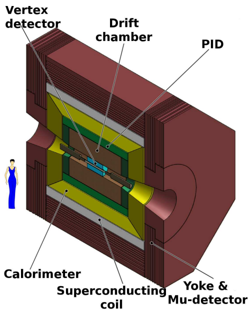


Planned integrated luminosity is 50 ab^{-1}

$$\sigma(b\bar{b}) = 1.05\ \text{nb} \quad N_{b\bar{b}} = 53 \times 10^9$$
$$\sigma(c\bar{c}) = 1.30\ \text{nb} \quad N_{c\bar{c}} = 65 \times 10^9$$
$$\boldsymbol{\sigma(\tau\tau) = 0.92\ \text{nb} \quad N_{\tau\tau} = 46 \times 10^9}$$

Introduction: Super Charm-Tau Factory





In five c.m.s. energy points ($2E = 3.554,\ 3.686,\ 3.770,\ 4.170,\ 4.650\,\mathrm{GeV}$) it is planned to accumulate $7\,\mathrm{ab}^{-1}$, which corresponds to $\boldsymbol{N_{\tau\tau} = 21 \times 10^9}$, which is 2.2 times smaller than the planned $\tau\tau$ statistics at Belle II.
However, the crucial feature of the Super Charm-Tau Factory project, the **polarized electron beam** and **lower c.m.s. energies**, might give some advantages in τ lepton studies in comparison with Belle II, thus, compensating smaller statistics of taus.

Precision studies of τ at e^+e^- factories

- **Michel parameters in $\tau \to \ell\nu\nu$ (ρ, η, ξ, δ):**

  Belle: Systematic uncertainties are about $(1 \div 3)$%; arXiv:1409.4969

- **Study of the radiative leptonic decays $\tau \to \ell\nu\nu\gamma$:**

  BABAR: Measurement of $\mathcal{B}(\tau \to \ell\nu\nu\gamma)$; PRD 91, 051103(R) (2015)

  Belle: $\bar{\eta} = -1.3 \pm 1.5 \pm 0.8$, $\xi\kappa = 0.5 \pm 0.4 \pm 0.2$; arXiv:1709.08833

- **Study of the 5-lepton decays $\tau \to \ell\ell'^+\ell'^-\nu\nu$:**

  CLEO: $\mathcal{B}(\tau \to ee e\nu\nu) = (2.8 \pm 1.5) \times 10^{-5}$,

  $\mathcal{B}(\tau \to \mu ee\nu\nu) < 3.6 \times 10^{-5} (CL = 90\%)$; PRL 76, 2637 (1996)

  Belle: statistical uncertainties are about $(3 \div 5)$%; J. Phys. Conf. Ser. **912** (2017) no.1, 012002.

- **Lepton universality with $\tau \to \ell\nu\nu$ and $\tau \to h\nu$ (h=π,K):**

  BABAR **:** $\left(\frac{g_\mu}{g_e}\right)_\tau = 1.0036 \pm 0.0020$, $\left(\frac{g_\tau}{g_\mu}\right)_h = 0.9850 \pm 0.0054$; PRL 105, 051602 (2010)

- **Tau lifetime:**
  Belle: $\tau_\tau = (290.17 \pm 0.53(\text{stat}) \pm 0.33(\text{syst}))$ fs; PRL 112, 031801 (2014)
  BABAR(prelim.): $\tau_\tau = (289.40 \pm 0.91(\text{stat}) \pm 0.90(\text{syst}))$ fs; Nucl. Phys. B 144, 105 (2005)

- **Tau mass:**

  BES3: $m_\tau = (1776.91 \pm 0.12(\text{stat}) \pm {}^{0.10}_{0.13}(\text{syst}))$ MeV/c^2; PRD 90, 012001 (2014)

  KEDR: $m_\tau = (1776.81 \pm {}^{0.25}_{0.23}(\text{stat}) \pm 0.15(\text{syst}))$ MeV/c^2; JETPL 85, 347 (2007)

  Belle: $m_\tau = (1776.61 \pm 0.13(\text{stat}) \pm 0.35(\text{syst}))$ MeV/c^2; PRL 99, 011801 (2007)
  BABAR: $m_\tau = (1776.68 \pm 0.12(\text{stat}) \pm 0.41(\text{syst}))$ MeV/c^2; PRD 80, 092005 (2009)

Michel parameters

In the SM charged weak interaction is described by the exchange of $W^\pm$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.

The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{\substack{N=S,V,T \\ i,j=L,R}} g_{ij}^N \left[\bar{u}_i(l^-)\Gamma^N v_n(\bar{\nu}_l)\right] \left[\bar{u}_m(\nu_\tau)\Gamma_N u_j(\tau^-)\right],$$

$$\Gamma^S = 1,\ \Gamma^V = \gamma^\mu,\ \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$$

Ten couplings g_{ij}^N, in the SM the only non-zero constant is $g_{LL}^V = 1$

Four bilinear combinations of g_{ij}^N, which are called as Michel parameters (MP): ρ, η, ξ and δ appear in the energy spectrum of the outgoing lepton:

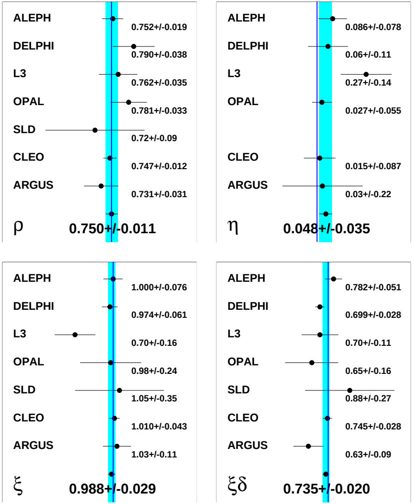
$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4}\sqrt{x^2 - x_0^2}\Bigg(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$\mp\frac{1}{3}P_\tau cos\theta_\ell \xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\Bigg),\ x = \frac{E_\ell}{E_{\max}},\ x_0 = \frac{m_\ell}{E_{\max}}$$

In the SM: $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$

Status of Michel parameters in τ decays

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.2%	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 2.8%	CLEO-97	3/4
ξ_h (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1



Status of Michel parameters in τ decays

With Belle statistics, which is about 300 times larger than the previous experimental $\tau\tau$ data samples, we can improve MP uncertainties by one order of magnitude.
In BSM models the couplings to τ are expected to be larger than those to μ. Contribution from New Physics in τ decays can be enhanced by a factor of $(\frac{m_\tau}{m_\mu})^2$.

- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\left(\frac{\tan^2\beta}{M^2_{H\pm}}\right)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:**
  $\mathcal{L} = \frac{g}{2\sqrt{2}} W^\mu \left\{ \bar{\nu}\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa^W_\tau}{2m_\tau}\partial^\nu\left(\bar{\nu}\sigma_{\mu\ nu}(1-\gamma^5)\tau\right)\right\}$,
  $-0.096 < \kappa^W_\tau < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB **701** (2011) 89
- **Heavy Majorana neutrino:** M. Doi *et al.*, Prog. Theor. Phys. **118** (2007) 1069.
- $\mu - \tau$ **LFV Yukawa couplings in** ξ_μ**:** K. Tobe, JHEP **1610** (2016) 114

Spin-dependent measurements with τ

To measure ξ and δ MP we have to know τ spin direction. At B factories, the effect of τ spin-spin correlation in $e^+e^- \to \tau^+(\vec{\zeta}^+)\tau^-(\vec{\zeta}^-)$ can be used.

At the Super Charm-Tau factory with polarized electron beam the average polarization of single τ is nonzero, hence the differental decay probability will contain both, τ spin-dependent and spin-independent parts.

$$\frac{d\sigma(\vec{\zeta}^-, \vec{\zeta}^+)}{d\Omega_\tau} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i^-\zeta_j^+ + \mathcal{P}_e(F_i^-\zeta_i^- + F_j^+\zeta_j^+))$$

$$D_0 = 1 + \cos^2\theta + \frac{1}{\gamma_\tau^2}\sin^2\theta, \; \mathcal{P}_e = \frac{N_e(+) - N_e(-)}{N_e(+) + N_e(-)}$$

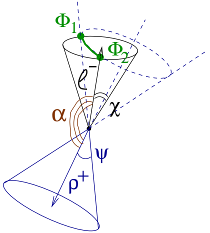
$$D_{ij} = \begin{pmatrix} (1+\frac{1}{\gamma_\tau^2})\sin^2\theta & 0 & \frac{1}{\gamma_\tau}\sin 2\theta \\ 0 & -\beta_\tau^2\sin^2\theta & 0 \\ \frac{1}{\gamma_\tau}\sin 2\theta & 0 & 1+\cos^2\theta - \frac{1}{\gamma_\tau^2}\sin^2\theta \end{pmatrix}$$

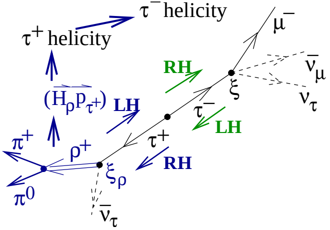
Single τ studies at the Super Charm-Tau factory:

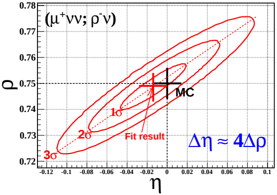
$$\frac{d\sigma(\vec{\zeta}^-)}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i^-\zeta_i^-)$$

At B factory: study of $(\ell\nu\nu;\ \rho\nu)$ and $(\rho\nu;\ \rho\nu)$ events

Effect of τ spin-spin correlation is used to measure ξ and δ MP.
Events of the $(\tau^{\mp} \to \ell^{\mp}\nu\nu;\ \tau^{\pm} \to \rho^{\pm}\nu)$ topology are used to measure: ρ, η, $\xi_\rho\xi$ and $\xi_\rho\xi\delta$, while $(\tau^{\mp} \to \rho^{\mp}\nu;\ \tau^{\pm} \to \rho^{\pm}\nu)$ events are used to extract ξ_ρ^2.







$$\frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho\xi A_3 + \xi_\rho\xi\delta A_4 = \sum_{i=0}^{4} A_i \Theta_i$$

$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^{\mp}\nu\nu,\rho^{\pm}\nu)}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} \left| \frac{\partial(E_\ell^*,\Omega_\ell^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)} \right| d\Phi_\tau$$

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}),\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z},\ \vec{\Theta} = (1,\rho,\eta,\xi_\rho\xi_\ell,\xi_\rho\xi_\ell\delta_\ell)$$

$$\boldsymbol{\mathcal{P}}_{\textbf{total}} = (1 - \sum_{i=1}^{4}\lambda_i)\boldsymbol{\mathcal{P}}_{signal}^{\boldsymbol{\ell-\rho}} + \lambda_1\mathcal{P}_{bg}^{\ell-3\pi} + \lambda_2\mathcal{P}_{bg}^{\pi-\rho} + \lambda_3\mathcal{P}_{bg}^{\rho-\rho} + \lambda_4\boldsymbol{\mathcal{P}}_{\boldsymbol{bg}}^{\boldsymbol{other}}\ \textbf{(MC)}$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space $\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi}^2,\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$ in CMS.

Method, $\tau^- \to h^-\nu_\tau$, $h = \pi,\ \rho$

$$J^\mu = < h|\bar{d}\gamma^\mu(c_V + c_A\gamma^5)u|0 >$$

Michel formalism for the $\tau^- \to h^-\nu_\tau$ includes:

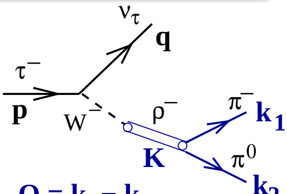
$$\xi_h = -\frac{2Re(c_V^* c_A)}{|c_V|^2+|c_A|^2} = -h_{\nu_\tau}(\text{ =1 in SM}):$$

$$\frac{d\Gamma(\tau^\mp \to \pi^\mp\nu)}{d\Omega_\pi} = C\left(1 \pm \xi_\pi P_\tau \cos\theta_\pi\right)$$

$$\frac{d\Gamma(\tau^\mp \to \rho^\mp\nu)}{dm^2_{\pi\pi}d\Omega_\rho d\Omega^*_\pi} = f(\vec{k}_1,\vec{k}_2) \pm \xi_\rho\vec{P}_\tau\vec{g}(\vec{k}_1,\vec{k}_2) = f(\vec{k}_1,\vec{k}_2)(1 \pm \xi_\rho\vec{P}_\tau\vec{H}_\rho)$$

$\vec{H}_\rho = M_\tau\frac{2(q,Q)\vec{Q}+Q^2\vec{K}}{2(p,Q)(q,Q)-Q^2(p,q)}$-polarimeter vector

Precision measurement of τ neutrino helisity, h_{ν_τ}, in various decay modes is an important test of the Standard Model.



Q = k₁ − k₂

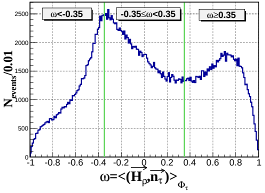
K = k₁ + k₂

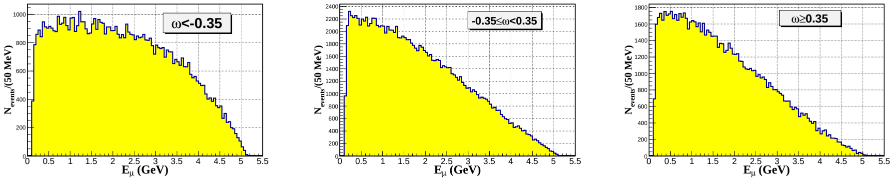
Method, helicity sensitive variable ω

M. Davier *et. al* Phys. Lett. B **306** (1993) 411.

Helicity sensitive variable ω is introduced as:

$$\omega = \frac{1}{\Phi_2 - \Phi_1} \int_{\Phi_1}^{\Phi_2} (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) d\Phi = < (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$$





Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Method, theoretical framework

- W. Fetscher, Phys. Rev. D **42** (1990) 1544.
  $\ell_1^{\mp} - \ell_2^{\pm}$, $\ell^{\mp} - h^{\pm}$, $\ell = e,\ \mu;\ h = \pi,\ K$.
- K. Tamai, Nucl. Phys. B **668** (2003) 385. (KEK Preprint 2003-14, Belle note 471) $\ell^{\mp} - \rho^{\pm}(\to \pi^{\pm}\pi^{0})$ + feasibility study.

$$\frac{d\sigma(\vec{\zeta},\vec{\zeta}')}{d\Omega} = \frac{\alpha^2}{64E_\tau^2}\beta_\tau(D_0 + D_{ij}\zeta_i\zeta_j')$$

$$\frac{d\Gamma(\tau^{\mp}(\vec{\zeta}^*) \to \ell^{\mp}\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi\vec{n}_\ell^*\vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\Gamma(\tau^{\pm}(\vec{\zeta'}^*) \to \rho^{\pm}\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* d\tilde{\Omega}_\pi} = \kappa_\rho(A' \mp \xi_\rho\vec{B'}\vec{\zeta'}^*)W(m_{\pi\pi}^2)$$

$$A' = 2(q,Q)Q_0^* - Q^2q_0^*,\ \vec{B'} = Q^2\vec{K}^* + 2(q,Q)\vec{Q}^*,\ W = |F_\pi(m_{\pi\pi}^2)|^2\frac{p_\rho(m_{\pi\pi}^2)\tilde{p}_\pi(m_{\pi\pi}^2)}{M_\tau m_{\pi\pi}}$$

$$\frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = \kappa_\ell\kappa_\rho\frac{\alpha^2\beta_\tau}{64E_\tau^2}(D_0A'A(E_\ell^*) + \xi_\rho\xi_\ell D_{ij}n_{\ell i}^*B_j'B(E_\ell^*))W(m_{\pi\pi}^2)$$

$$\frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2}\frac{d\sigma(\ell^{\mp},\rho^{\pm})}{dE_\ell^* d\Omega_\ell^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(E_\ell^*,\Omega_\ell^*,\Omega_\rho^*,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right|d\Phi_\tau$$

Multidimensional unbinned maximum likelihood fit

4 Michel parameters ($\vec{\Theta} = (1,\ \rho,\ \eta,\ \xi_\rho\xi_\ell,\ \xi_\rho\xi_\ell\delta_\ell)$) are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space in CMS,
$\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$. The PDF for individual k-th event is written in the form:

$$\mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})},\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z}$$

Likelihood function for N events:

$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{L} = -\ln L = N \ln \mathcal{N}(\vec{\Theta}) - \sum_{k=1}^{N} \ln \mathcal{F}^{(k)},\ \mathcal{F}^{(k)} = \mathcal{F}(\vec{z}^{(k)})$$

$$\mathcal{F}^{(k)} = A_0^{(k)}\Theta_0 + A_1^{(k)}\Theta_1 + A_2^{(k)}\Theta_2 + A_3^{(k)}\Theta_3 + A_4^{(k)}\Theta_4 = \sum_{i=0}^{4} A_i^{(k)}\Theta_i$$

$$\mathcal{N} = C_0\Theta_0 + C_1\Theta_1 + C_2\Theta_2 + C_3\Theta_3 + C_4\Theta_4,\ C_j = \frac{1}{N}\sum_{k=1}^{N} C_j^{(k)},\ C_j^{(k)} = \frac{A_j^{(k)}}{\sum\limits_{i=0}^{4} A_i^{(k)}\Theta_i^{MC}}$$

$$\vec{\Theta}^{MC} = (1,\ 0.75,\ 0,\ 1,\ 0.75),\ \mathcal{L} = N \ln\left(\sum_{j=0}^{4} C_j\Theta_j\right) - \sum_{k=1}^{N} \ln\left(\sum_{i=0}^{4} A_i^{(k)}\Theta_i\right)$$

As a result fitted statistics is represented by a set of $5 \times N$ values of $A_i^{(k)}$ $(k = 1 \div N,\ i = 0 \div 4)$, which is calculated only once.

C_i $(i = 0 \div 4)$ are calculated using MC simulation.

In ideal case (no rad. corr., $\varepsilon = 100\%$): $C_0 = 1$, $C_2 = 4m_\ell/m_\tau$, $C_{1,3,4} = 0$

Suppose we have N_{MC} MC events, which were simulated with particular set $\vec{\Theta}^{\mathrm{MC}}$. By reweighting each event we can calculate normalization for arbitrary set $\vec{\Theta}$:

$$\mathcal{N}(\vec{\Theta}) \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} w^{(k)},\ w^{(k)} = \frac{A_i^{(k)}\Theta_i}{A_j^{(k)}\Theta_j^{MC}} = B_m^{(k)}\Theta_m,\ B_m^{(k)} = \frac{A_m^{(k)}}{A_j^{(k)}\Theta_j^{MC}}$$

$$\mathcal{N}(\vec{\Theta}) = C_i\Theta_i,\ C_i = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} B_i^{(k)}$$

This algorithm can be easily extended to take into account selection efficiency:

$$\mathcal{F}(\vec{z}) \to \mathcal{F}'(\vec{z}) = \mathcal{F}(\vec{z})\epsilon(\vec{z}),\ \mathcal{N}'(\vec{\Theta}) = \int \mathcal{F}(\vec{z})\epsilon(\vec{z})d\vec{z}$$

$$\mathcal{L} = N_{\mathrm{sel}} \ln \mathcal{N}'(\vec{\Theta}) - \sum_{k=1}^{N_{\mathrm{sel}}} \ln(\mathcal{F}^{(k)}\epsilon(\vec{z})) = N_{\mathrm{sel}} \ln(C_i'\Theta_i) - \sum_{k=1}^{N_{\mathrm{sel}}} \ln(A_i^{(k)}\Theta_i) - \sum_{k=1}^{N_{\mathrm{sel}}} \ln\epsilon(\vec{z})$$

$$C_i' = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}^{\mathrm{sel}}} B_i^{(k)}$$

Accuracy of the evaluation of the C_i' coefficients is crucial in the precision measurement of Michel parameters.

Corrections, detector effects, background

Physical corrections:

- All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included
- Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$
- Radiative decay $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$

Detector effects:

- Track momentum resolution
- γ energy and angular resolution
- Effect of external bremsstrahlung for $e-\rho$ events
- Beam energy spread
- EXP/MC efficiency corrections (trigger, track rec., π^0 rec., ℓID, πID)

Background:

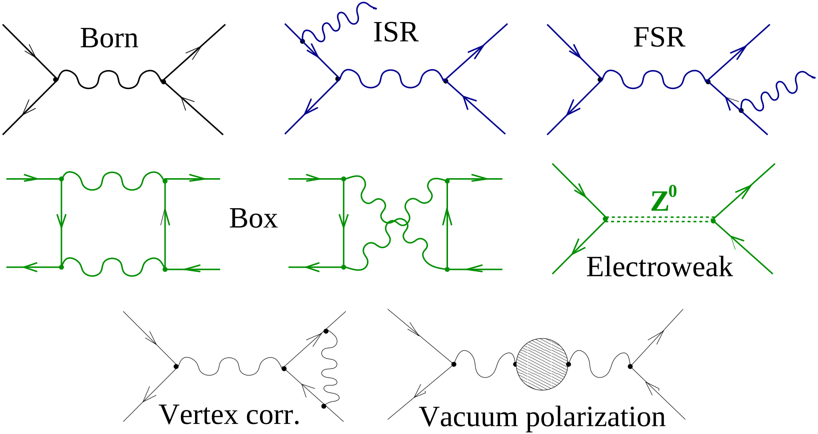
The main background comes from $(\ell\nu\nu;\ \pi 2\pi^0\nu)$(∼10%), $(\pi\nu;\ \pi\pi^0\nu)$(∼1.5%) and $(\rho^+\nu;\ \rho^-\nu)$(∼0.5%) events, it is included in PDF analytically. The remaining background(∼2.0%) is taken into account using MC-based approach.

Background from the non-$\tau\tau$ events is $\lesssim 0.1\%$.

Physical corrections

- **Radiative corrections to $e^+e^- \to \tau^+\tau^-$**
  - All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included:
    S. Jadach and Z. Was, Acta Phys. Polon. B **15** (1984) 1151 [Erratum-ibid. B **16** (1985) 483].
    A. B. Arbuzov *et al* JHEP **9710** (1997) 001.
  - KKMC based approach:
    We generate table of ISR photons and then use it to calculate visible differential cross section in CMS.
- **Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$**
  - Analytical approach based on:
    A. B. Arbuzov, Phys. Lett. B **524** (2002) 99. $\mathcal{O}(\alpha)$.
    A. Arbuzov, A. Czarnecki and A. Gaponenko, Phys. Rev. D **65** (2002) 113006. $\mathcal{O}(\alpha^2 \ln^2(\frac{m_\mu}{m_e}))$.
    A. Arbuzov and K. Melnikov, Phys. Rev. D **66** (2002) 093003. $\mathcal{O}(\alpha^2 \ln(\frac{m_\mu}{m_e}))$.
  - TAUOLA based approach:
    M. Jezabek, Comput. Phys. Commum. **70** (1992) 69.
    A. Czarnecki, M. Jezabek and J. H. Kuhn, Nucl. Phys. B **351** (1991) 70.
- **Radiative corrections to $\tau^- \to \pi^-\pi^0\nu_\tau$**
  - Analytical approach based on:
    F. Flores-Baez *et al*, Phys. Rev. Lett. D **74** (2006) 071301(R).
    A. Flores-Tlalpa *et al*, Nucl. Phys. B (Proc. Suppl.) **169** (2007) 250.
  - PHOTOS based approach

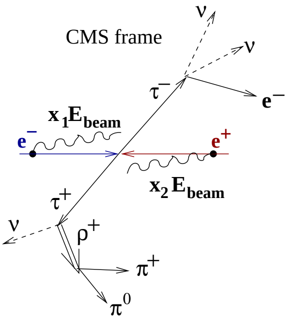
$\mathcal{O}(\alpha^3)$ corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$

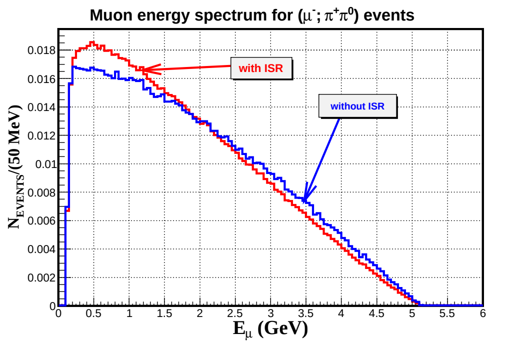


S. Jadach and Z. Was, Acta Phys. Polon. B **15** (1984) 1151 [Erratum-ibid. B **16** (1985) 483].
A. B. Arbuzov *et al* JHEP **9710** (1997) 001.

Charge-odd part of the cross section comes from the interference of the ISR and FSR diagrams as well as box and Born diagrams, and Z^0-exchange and Born diagrams.

Initial state radiation (ISR)





$$\frac{d\sigma_{\rm vis}(s)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} = \iint_0^1 dx_1 dx_2 D(x_1) D(x_2) \frac{d\sigma(s(1-x_1)(1-x_2))}{dp'_\ell d\Omega'_\ell dp'_\rho d\Omega'_\rho dm^2_{\pi\pi} d\tilde{\Omega}_\pi} \left|\frac{\partial(p'_\ell, \Omega'_\ell)}{\partial(p_\ell, \Omega_\ell)}\right| \left|\frac{\partial(p'_\rho, \Omega'_\rho)}{\partial(p_\rho, \Omega_\rho)}\right|$$

- $D(x) = x^{\beta/2-1}h(x)$ - probability function for initial $e^\mp$ to emit a γ-quantum jet carrying $x_{1,2}$ part of $e^\mp$ energy $E_{\rm beam} = \sqrt{s}/2$. $\beta = \frac{2\alpha}{\pi}(\ln\frac{s}{m^2} - 1)$, $h(x)$ - smooth limited function.
- $\left|\frac{\partial(p'_i, \Omega'_i)}{\partial(p_i, \Omega_i)}\right|$ $(i = \ell,\ \rho)$ - Jacobian of transformation from the $\tau^+\tau^-$ rest frame to the Belle CMS.

At the Super Charm-Tau factory the impact of the ISR is expected to be essentially smaller.

Description of background

Total PDF

$$\mathcal{P}(x) = \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}}\Bigg((1-\sum_i \lambda_i)\frac{S(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} S(x)dx} + \lambda_{3\pi}\frac{\tilde{B}_{3\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{3\pi}(x)dx} + \lambda_{\pi}\frac{\tilde{B}_{\pi}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{\pi}(x)dx} + \lambda_{\rho}\frac{\tilde{B}_{\rho}(x)}{\int \frac{\overline{\varepsilon(x)}}{\bar{\varepsilon}} \tilde{B}_{\rho}(x)dx} +$$

$$+(1-\sum_i \lambda_i)\frac{N^{\rm sel}_{\rm rest}(x)}{N^{\rm sel}_{\rm sig}(x)} S_{\rm SM}(x)\Bigg)$$

$$\tilde{B}_{3\pi}(x) = \int 2(1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_{3\pi}(x,y)dy,\ \tilde{B}_{\pi}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}B_{\pi}(x)$$

$$\tilde{B}_{\rho}(x) = \frac{\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)}{\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)}\int (1-\varepsilon_{\pi^0}(y))\varepsilon_{\rm add}(y)B_{\rho}(x,y)dy,\ \overline{\varepsilon(x)} = \epsilon^{\rm EXP}_{\rm corr}(x)\varepsilon(x)$$

- $x = (p_\ell,\ \Omega_\ell,\ p_\rho,\ \Omega_\rho,\ m^2_{\pi\pi},\ \tilde{\Omega}_\pi);\ y = (p_{\pi^0},\ \Omega_{\pi^0});$
- $S(x)$ - theoretical density of signal $(\ell^\mp\nu\nu,\ \rho^\pm\nu)$ events;
- $B_{3\pi}(x,y)$ - theoretical density of background $(\ell^\mp\nu\nu,\ \pi^\pm 2\pi^0\nu)$ events;
- $B_\pi(x)$ - theoretical density of background $(\pi^\mp\nu,\ \rho^\pm\nu)$ events;
- $B_\rho(x)$ - theoretical density of background $(\rho^\mp\nu,\ \rho^\pm\nu)$ events;
- $\varepsilon(x)$ - detection efficiency for signal events (**common multiplier**);
- $N^{\rm sel}_{\rm rest}(x)/N^{\rm sel}_{\rm sig}(x)$ - number of the selected (remaining/signal) MC events in the multidimensional cell around "x". Admixture of the remaining background is $(1\div 2)$%.
- λ_i - i-th background fraction (from MC)
- $\varepsilon_{\pi^0}(y)$ - π^0 detection efficiency (tabulated from MC);
- $\varepsilon_{\rm add}(y) = \varepsilon^{3\pi}_{\rm add}(y)/\varepsilon^{\rm sig}_{\rm add}$ - ratio of the $E^{\rm LAB}_{\gamma\rm rest}$ cut efficiencies (tabulated from MC);
- $\varepsilon^{\mu ID}_{\pi\to\mu}(p_\ell,\ \Omega_\ell)/\varepsilon^{\mu ID}_{\mu\to\mu}(p_\ell,\ \Omega_\ell)$ is tabulated from MC;
- $\epsilon^{\rm EXP}_{\rm corr}(x)$ - EXP/MC efficiency correction.

Systematic uncertainties

Source	$\Delta(\rho)$, %	$\Delta(\eta)$, %	$\Delta(\xi_\rho\xi)$, %	$\Delta(\xi_\rho\xi\delta)$, %
		Physical corrections		
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
		Apparatus corrections		
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\text{beam}})$	0.07	0.25	0.03	0.15
		Normalization		
$\Delta\mathcal{N}$	0.11	0.50	0.17	0.13
without EXP/MC corr.	**0.29**	**0.95**	**0.38**	**0.38**
$\mathcal{R}_{\text{trg}}$	~ 1	~ 2	~ 3	~ 3

Super Charm-Tau factory, $\tau^- \to \ell^- \bar{\nu}_\ell \nu_\tau$

$$\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i \zeta_i)$$

$$\frac{d\Gamma(\tau^\mp(\vec{\zeta}^*) \to \ell^\mp\nu\nu)}{dx^* d\Omega_\ell^*} = \kappa_\ell(A(x^*) \mp \xi_\ell \vec{n}_\ell^* \vec{\zeta}^* B(x^*)),\ x^* = E_\ell^*/E_{\ell max}^*$$

$$A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*),\ B(x^*) = B_1(x^*) + \delta B_2(x^*)$$

$$\frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau} = \kappa_\ell \frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 A(E_\ell^*) \mp \mathcal{P}_e \xi_\ell F_i n_{\ell i}^* B(E_\ell^*))$$

$$\frac{d\sigma(\ell^\mp)}{dp_\ell d\Omega_\ell} = \int\limits_{\Omega_\tau-\mathrm{sector}} \frac{d\sigma(\ell^\mp)}{dE_\ell^* d\Omega_\ell^* d\Omega_\tau}\left|\frac{\partial(E_\ell^*, \Omega_\ell^*)}{\partial(p_\ell, \Omega_\ell)}\right| d\Omega_\tau$$

Ω_τ-sector is determined by the kinematical constraint $m_{\nu\nu} > 0$

All Michel parameters (ρ, η, $\mathcal{P}_e\xi$, $\mathcal{P}_e\xi\delta$) are measured in the unbinned maximum likelihood fit of ($\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau$; $\tau^+ \to \mathrm{all}$) events in the **3D** phase space. Due to the unideal detection efficiency for the decays of the opposite tau, there is still some contribution from the spin-spin correlation term.

The reduced 3D phase space allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.

Super Charm-Tau factory, $\tau^- \to \pi^-/\rho^-\nu_\tau$

$$\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E_\tau^2}\beta_\tau(D_0 + \mathcal{P}_e F_i \zeta_i)$$

$$\frac{d\Gamma(\tau^\mp \to \pi^\mp\nu)}{d\Omega_\pi^*} = \kappa_\pi(1 \pm \xi_\pi \vec{\zeta}\vec{n}_\pi^*), \quad \frac{d\Gamma(\tau^\mp \to \rho^\mp\nu)}{dm_{\pi\pi}^2 d\Omega_\rho^* \tilde{\Omega}_\pi} = f(\vec{k}_1, \vec{k}_2)(1 \pm \xi_\rho \vec{\zeta}\vec{H}_\rho^*)$$

$$\frac{d\sigma(\pi^\mp)}{d\Omega_\pi^* d\Omega_\tau} = \kappa_\pi \frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 \pm \mathcal{P}_e \xi_\pi F_i n_{\pi i}^*)$$

$$\frac{d\sigma(\rho^\mp)}{d\Omega_\rho^* dm_{\pi\pi}^2 \tilde{\Omega}_\pi d\Omega_\tau} = f(\vec{k}_1, \vec{k}_2)\frac{\alpha^2\beta_\tau}{32E_\tau^2}(D_0 \pm \mathcal{P}_e \xi_\rho F_i H_{\rho i}^*)$$

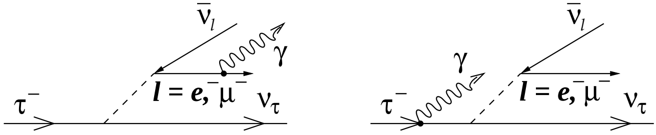
$$\frac{d\sigma(\pi^\mp)}{dp_\pi d\Omega_\pi} = \int_0^{2\pi} \frac{d\sigma(\pi^\mp)}{d\Omega_\pi^* d\Omega_\tau}\left|\frac{\partial(\Omega_\pi^*, \Omega_\tau)}{\partial(p_\pi, \Omega_\pi, \Phi_\tau)}\right| d\Phi_\tau$$

$$\frac{d\sigma(\rho^\mp)}{dp_\rho d\Omega_\rho dm_{\pi\pi}^2 \tilde{\Omega}_\pi} = \int_0^{2\pi} \frac{d\sigma(\rho^\mp)}{d\Omega_\rho^* dm_{\pi\pi}^2 \tilde{\Omega}_\pi d\Omega_\tau}\left|\frac{\partial(\Omega_\rho^*, \Omega_\tau)}{\partial(p_\rho, \Omega_\rho, \Phi_\tau)}\right| d\Phi_\tau$$

Parameters ($\mathcal{P}_e\xi_\pi$, $\mathcal{P}_e\xi_\rho$) are measured in the unbinned maximum likelihood fit of the ($\tau^- \to \pi^-/\rho^-\nu_\tau$; $\tau^+ \to \mathrm{all}$) events. These decays can be used to monitor $\mathcal{P}_e$ with high precision.

Michel parameters in $\tau \to \ell\nu\nu\gamma$, $(\ell = e, \mu)$ (I)

C. Fronsdal and H. Uberall, Phys. Rev. **113** (1959) 654. ($m_\ell = 0$)
A. B. Arbuzov and T. V. Kopylova, JHEP **1609** (2016) 109. ($m_\ell \neq 0$)



Photon carries information about spin state of outgoing lepton, as a result two additional parameters, $\bar{\eta}$ and $\xi\kappa$, can be extracted.
These parameters were measured in τ decays at Belle for the first time.

$$\frac{d\Gamma(\tau^\mp \to \ell^\mp \nu_\ell \nu_\tau \gamma)}{dx\, dy\, d\Omega_\ell\, d\Omega_\gamma} = \Gamma_0 \frac{\alpha}{64\pi^3} \frac{\beta_\ell}{y} \left[F(x,y,d) \pm P_\tau \left(\beta_\ell \cos\theta_\ell G(x,y,d) + \cos\theta_\gamma H(x,y,d)\right)\right],$$

$$\Gamma_0 = G_F^2 m_\tau^5/192\pi^3,\ \beta_\ell = \sqrt{1 - m_\ell^2/E_\ell^2},\ x = 2E_\ell/m_\tau,\ y = 2E_\gamma/m_\tau,\ d = 1 - \beta_\ell \cos\theta_{\ell\gamma}$$

$$F = F_0 + \bar{\eta} F_1,\ G = G_0 + \xi\kappa G_1,\ H = H_0 + \xi\kappa H_1,\ \frac{d\sigma(\ell^\mp\nu\nu\gamma, \rho^\pm\nu)}{dE_\ell^* d\Omega_\ell^* dE_\gamma^* d\Omega_\gamma^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} = A_0 + \bar{\eta} A_1 + \xi\kappa A_2$$

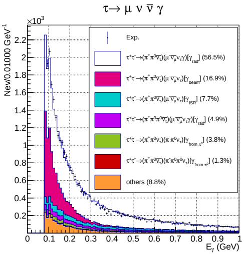
$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp\nu\nu\gamma, \rho^\pm\nu)}{dp_\ell d\Omega_\ell dp_\gamma d\Omega_\gamma dp_\rho d\Omega_\rho dm_{\pi\pi}^2 d\tilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp\nu\nu\gamma, \rho^\pm\nu)}{dE_\ell^* d\Omega_\ell^* dE_\gamma^* d\Omega_\gamma^* d\Omega_\rho^* dm_{\pi\pi}^2 d\tilde{\Omega}_\pi d\Omega_\tau} |\mathrm{JACOBIAN}|\, d\Phi_\tau$$

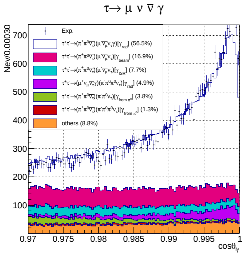
$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})} = \frac{\mathcal{F}_0 + \mathcal{F}_1 \bar{\eta} + \mathcal{F}_2 \xi\delta}{\mathcal{N}_0 + \mathcal{N}_1 \bar{\eta} + \mathcal{N}_2 \xi\delta},\ \mathcal{N}_k = \int \mathcal{F}_k(\vec{z}) d\vec{z},\ (k = 0, 1, 2)$$

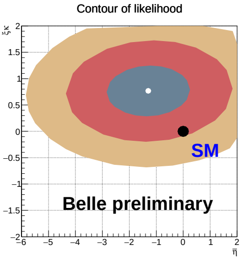
$\bar{\eta}$ and $\xi\delta$ are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu\gamma;\ \rho\nu)$ events in the 12D phase space in CMS.

Michel parameters in $\tau \to \ell\nu\nu\gamma$, ($\ell = e, \mu$) (II)

$N_{\tau\tau} = 646 \times 10^6$, **selected: 71171** $(\mu\nu\nu\gamma;\ \rho\nu)$ **and 776834** $(e\nu\nu\gamma;\ \rho\nu)$ **events**







Source	$\sigma^e_{\bar\eta}$	$\sigma^e_{\xi\kappa}$	$\sigma^\mu_{\bar\eta}$	$\sigma^\mu_{\xi\kappa}$
Normalization	4.3	0.94	0.15	0.04
Background PDF	2.5	0.24	0.67	0.22
Branching ratios	3.8	0.05	0.25	0.01
Cluster merge in ECL	2.2	0.46	0.02	0.06
Detector resolution	0.74	0.20	0.22	0.02
Data/MC eff. corr.	1.9	0.14	0.04	0.04
Total	7.0	1.1	0.76	0.24

Belle preliminary

$$\bar\eta = -\mathbf{1.3} \pm \mathbf{1.5} \pm \mathbf{0.8}$$

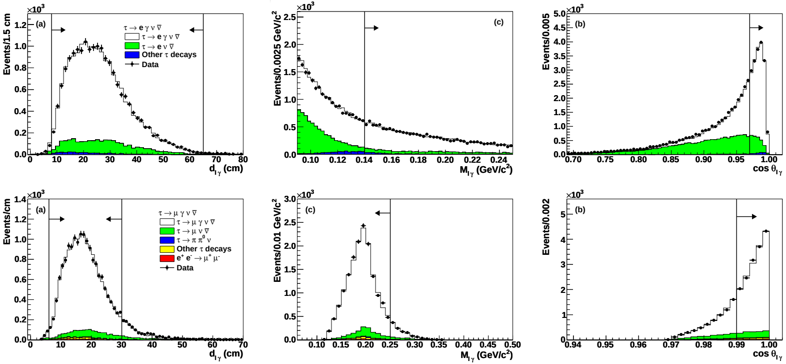
$$\xi\kappa = \mathbf{0.5} \pm \mathbf{0.4} \pm \mathbf{0.2}$$

Measurement of $\mathcal{B}(\tau \to \ell\nu\nu\gamma)$ at *BABAR* (I)

$$\int Ldt = 431\ \mathrm{fb}^{-1}$$

Selections:

- 2-track events with zero net charge and 1 photon with $E_\gamma > 50$ MeV;
- 0.9<thrust<0.995, signal hemisphere: $\ell + \gamma$, tag hemisphere: track+neutrals;
- reject $\ell^\mp - \ell^\pm$ events, $E_{\mathrm{tot}} < 9$ GeV, distance between track and photon clusters $d_{\ell\gamma} < 100$ cm.



$e\nu\nu\gamma$	$0.22 \le E_\gamma \le 2.0\,$GeV, $M_{e\gamma} \ge 0.14\,$GeV/c^2, $\cos\theta_{e\gamma} \ge 0.97$, $8 \le d_{e\gamma} \le 65\,$cm
$\mu\nu\nu\gamma$	$0.10 \le E_\gamma \le 2.5\,$GeV, $M_{\mu\gamma} \le 0.25\,$GeV/c^2, $\cos\theta_{\mu\gamma} \ge 0.99$, $6 \le d_{\mu\gamma} \le 30\,$cm

$$N_{\mathrm{sel}}(\mu\nu\nu\gamma) = 15688 \pm 125 \quad N_{\mathrm{sel}}(e\nu\nu\gamma) = 18149 \pm 135$$

Measurement of $\mathcal{B}(\tau \to \ell\nu\nu\gamma)$ at *BABAR* (II)

$$\mathcal{B} = \frac{N_{\rm sel}(1 - f_{\rm bg})}{2\sigma_{\tau\tau}\mathcal{L}\varepsilon}$$

	$\mu\nu\nu\gamma$	$e\nu\nu\gamma$
ε (%)	0.480 ± 0.010	0.105 ± 0.003
$f_{\rm bg}$	0.102 ± 0.002	0.156 ± 0.003

	$\tau \to \mu\nu\nu\gamma$	$\tau \to e\nu\nu\gamma$
Photon efficiency	1.8	1.8
Particle identification	1.5	1.5
Background evaluation	0.9	0.7
BF	0.7	0.7
Luminosity and cross section	0.6	0.6
MC statistics	0.5	0.6
Selection criteria	0.5	0.5
Trigger selection	0.5	0.6
Track reconstruction	0.3	0.3
Total	2.8	2.8

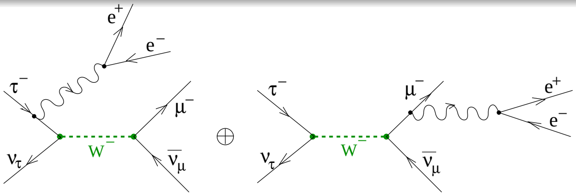
$$\mathcal{B}(\tau \to \mu\nu\nu\gamma)[E^*_\gamma > 10\,\mathrm{MeV}] = (3.69 \pm 0.03 \pm 0.10) \times 10^{-3}$$

$$\mathcal{B}(\tau \to e\nu\nu\gamma)[E^*_\gamma > 10\,\mathrm{MeV}] = (1.847 \pm 0.015 \pm 0.052) \times 10^{-2}$$

Measured branching ratios agree with the LO predictions ($\mathcal{B}(\mu\nu\nu\gamma) = 3.663 \times 10^{-3}$, $\mathcal{B}(e\nu\nu\gamma) = 1.834 \times 10^{-2}$), however the LO+NLO prediction for the $\tau \to e\nu\nu\gamma$ ($\mathcal{B}(e\nu\nu\gamma) = 1.645 \times 10^{-2}$) differs from the experimental result by **3.5σ**. It is important to embed NLO corrections to the MC generator (TAUOLA) of the radiative leptonic decay. Also background from the doubly-radiative leptonic decays should be properly studied and subtracted.

M. Fael, L. Mercolli and M. Passera, JHEP 1507 (2015) 153.

Tau decays into 5 leptons (I)



D. A. Dicus and R. Vega, Phys. Lett. B **338** (1994) 341.

M. S. Alam *et al.* [CLEO Collaboration], Phys. Rev. Lett. **76** (1996) 2637.

A. Flores-Tlalpa, G. Lopez Castro and P. Roig, JHEP 1604 (2016) 185.

Mode	$\mathcal{B}_{\rm theory}$	$\mathcal{B}_{\rm CLEO}$
$e^\mp e^+ e^- 2\nu$	$(4.21 \pm 0.01) \times 10^{-5}$	$(2.7^{+1.6}_{-1.2}) \times 10^{-5}$
$\mu^\mp e^+ e^- 2\nu$	$(1.984 \pm 0.004) \times 10^{-5}$	$< 3.2 \times 10^{-5}$(90% CL)
$e^\mp \mu^+ \mu^- 2\nu$	$(1.247 \pm 0.001) \times 10^{-7}$	
$\mu^\mp \mu^+ \mu^- 2\nu$	$(1.183 \pm 0.001) \times 10^{-7}$	

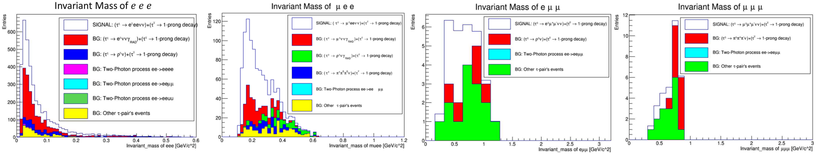
A. Kersch, N. Kraus and R. Engfer [SINDRUM], Nucl. Phys. A **485** (1988) 606.

$$\frac{d\Gamma(\tau)}{d\mathcal{PS}} = Q_{LL}d_1 + Q_{LR}d_2 + Q_{RL}d_3 + Q_{RR}d_4 + B_{RL}d_5 + B_{LR}d_6$$

Up to now Q_{LL}, Q_{LR}, Q_{RL}, Q_{RR}, B_{RL}, B_{LR} were measured only in muon decays ($\mu^- \to e^- e^- e^+ \nu_\mu \bar{\nu}_e$) with the accuracy of about $10 \div 20\%$.

Recently, analysis of 5-lepton τ decays has been started at Belle.

Tau decays into 5 leptons (II)



τ^- decay mode	$e^-e^+e^-\bar{\nu}_e\nu_\tau$	$\mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$	$e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$	$\mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$
Detection efficiency, %	1.769±0.004	1.204±0.003	3.561±0.006	1.674±0.004
Main background(s)	$e^-\bar{\nu}_e\nu_\tau\gamma$, $\pi^-\pi^0\nu_\tau$	$\mu^-\bar{\nu}_\mu\nu_\tau\gamma$, $\pi^-\pi^0(\to e^+e^-\gamma)\nu_\tau$, $\pi^-\pi^0\pi^0\nu_\tau$	$\pi^-\pi^0\nu_\tau$	$\pi^-\pi^+\pi^-\nu_\tau$
Expected number of signal events	1300	430	8	4
Fraction of the signal, %	47	50	37	16

The study is performed as a blinded analysis. Selection criteria were elaborated. The expected background in the signal region is estimated. Systematic uncertainties are under investigation.

Tau decays into 5 leptons (III)

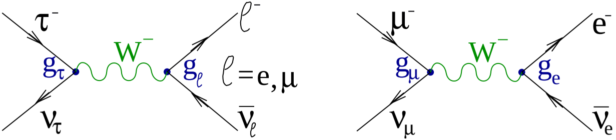
- Michel parameters can be measured in two ways: in the study of the dynamics and from the measurement of the branching fraction:

  $(\mathcal{B}_{\rm exp} - \mathcal{B}_{\rm SM})/\mathcal{B}_{\rm SM} = (Q_{LL} - 1) + \alpha_{LR} Q_{LR} + \alpha_{RL} Q_{RL} + \alpha_{RR} Q_{RR} + \beta_{RL} B_{RL} + \beta_{LR} B_{LR}$

- Recently, the possibility to measure anomalous magnetic moment of τ, a_τ was discussed in arXiv:1711.01393
  $\mathcal{B} = \mathcal{B}_0 + a_\tau \mathcal{B}_1$.

Lepton universality in the SM

$$g_e = g_\mu = g_\tau$$



$$\Gamma(L^- \to \ell^- \bar\nu_\ell \nu_L(\gamma)) = \frac{\mathcal{B}(L^- \to \ell^- \bar\nu_\ell \nu_L(\gamma))}{\tau_L} = \frac{g_L^2 g_\ell^2}{32 M_W^4} \frac{m_L^5}{192\pi^3} F_{\rm corr}(m_L, m_\ell)$$

$$F_{\rm corr}(m_L, m_\ell) = f(x)\left(1 + \frac{3}{5}\frac{m_L^2}{M_W^2}\right)\left(1 + \frac{\alpha(m_L)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right)$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \; x = m_\ell / m_L$$

$$\mathcal{B}(\mu^- \to e^- \bar\nu_e \nu_\mu(\gamma)) = 1$$

$$\frac{g_\tau}{g_e} = \sqrt{\mathcal{B}(\tau^- \to \mu^- \bar\nu_\mu \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\rm corr}(m_\mu, m_e)}{F_{\rm corr}(m_\tau, m_\mu)}}, \quad \frac{\mathbf{g}_\tau}{\mathbf{g_e}} = \mathbf{1.0029 \pm 0.0015} \text{ (HFAG2017)}$$

$$\frac{g_\tau}{g_\mu} = \sqrt{\mathcal{B}(\tau^- \to e^- \bar\nu_e \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} \frac{F_{\rm corr}(m_\mu, m_e)}{F_{\rm corr}(m_\tau, m_e)}}, \quad \frac{\mathbf{g}_\tau}{\mathbf{g}_\mu} = \mathbf{1.0010 \pm 0.0015} \text{ (HFAG2017)}$$

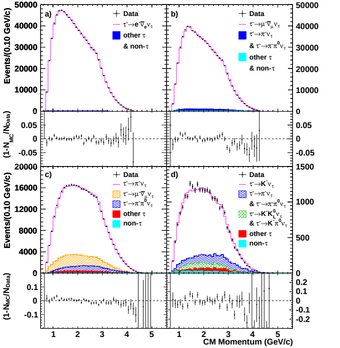
$$\frac{g_\mu}{g_e} = \sqrt{\frac{\mathcal{B}(\tau^- \to \mu^- \bar\nu_\mu \nu_\tau(\gamma))}{\mathcal{B}(\tau^- \to e^- \bar\nu_e \nu_\tau(\gamma))} \frac{F_{\rm corr}(m_\tau, m_e)}{F_{\rm corr}(m_\tau, m_\mu)}}, \quad \frac{\mathbf{g}_\mu}{\mathbf{g_e}} = \mathbf{1.0019 \pm 0.0014} \text{ (HFAG2017)}$$

Test of lepton universality at *BABAR* (I)

$$\int L dt = 467\ \mathrm{fb}^{-1}$$

Selections:

- 4-track events with zero net charge;
- $0.1\sqrt{s} < E^{\mathrm{CMS}}_{\mathrm{miss}} < 0.7\sqrt{s}$, $|\cos(\theta^{\mathrm{CMS}}_{\mathrm{miss}})| < 0.7$
- thrust> 0.9, signal hemisphere: $\ell/h(\ell = e, \mu;\ h = \pi, K)$, tag hemisphere: $\tau \to \pi\pi\pi\nu$;
- signal hemisphere: $E^{LAB}_{\mathrm{extra}\gamma} < \{1.0,\ 0.5,\ 0.2,\ 0.2\}$ GeV for $\{e,\ \mu,\ \pi,\ K\}$, respectively



	μ	π	K
$\mathbf{N}^{\mathrm{D}}$	731102	369091	25123
Purity	97.3%	78.7%	76.6%
Total Efficiency	0.485%	0.324%	0.330%
Particle ID Efficiency	74.5%	74.6%	84.6%
Systematic uncertainties:			
Particle ID	0.32	0.51	0.94
Detector response	0.08	0.64	0.54
Backgrounds	0.08	0.44	0.85
Trigger	0.10	0.10	0.10
$\pi^-\pi^-\pi^+$ modelling	0.01	0.07	0.27
Radiation	0.04	0.10	0.04
$\mathcal{B}(\tau^- \to \pi^-\pi^-\pi^+\nu_\tau)$	0.05	0.15	0.40
$\mathcal{L}\sigma_{\tau\tau}$	0.02	0.39	0.20
Total [%]	**0.36**	**1.0**	**1.5**

$\boldsymbol{\tau \to e\nu\nu}$: $N_{\mathrm{sel}} = 884426$, $\varepsilon = (0.589 \pm 0.010)$%, purity is (99.69 ± 0.06)%

Test of lepton universality at *BABAR* (II)

$$R_\mu = \frac{\mathcal{B}(\tau \to \mu\nu\nu)}{\mathcal{B}(\tau \to e\nu\nu)} = 0.9796 \pm 0.0016 \pm 0.0036$$

$$R_\pi = \frac{\mathcal{B}(\tau \to \pi\nu)}{\mathcal{B}(\tau \to e\nu\nu)} = 0.5945 \pm 0.0014 \pm 0.0061$$

$$R_K = \frac{\mathcal{B}(\tau \to K\nu)}{\mathcal{B}(\tau \to e\nu\nu)} = 0.03882 \pm 0.00032 \pm 0.00057$$

$$\left(\frac{g_\mu}{g_e}\right)_\tau = \sqrt{R_\mu \frac{F_{\rm corr}(m_\tau, m_e)}{F_{\rm corr}(m_\tau, m_\mu)}} = 1.0036 \pm 0.0020$$

$$\left(\frac{g_\tau}{g_\mu}\right)^2_h = \frac{\mathcal{B}(\tau \to h\nu_\tau)}{\mathcal{B}(h \to \mu\nu_\mu)} \frac{2m_h m_\mu^2 \tau_h}{(1+\delta_h) m_\tau^3 \tau_\tau} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2$$

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9856 \pm 0.0057, \quad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9827 \pm 0.0086$$

$$\left(\frac{g_\tau}{g_\mu}\right)_h = 0.9850 \pm 0.0054$$ (2.8σ away from SM)

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 1.0000 \pm 0.0014$$ (HFAG2017)

Summary

- **The world largest statistics of τ leptons collected by Belle and *BABAR* opens new era in the precision tests of the Standard Model and search for the effects of New Physics.**
- Complementary study of leptonic τ decays at *BABAR* and Belle. *BABAR* measured precisely the ratio of the leptonic branching ratios to test lepton universality. While Belle is working on the precision measurement of Michel parameters.
- Nonzero average polarization of single τ at the Super Charm-Tau factory provides the possibility to measure all Michel parameters without tagging the opposite tau. Better systematic uncertainty can be reached due to the smaller impact of the ISR as well as smaller number of PS dimensions. Effect of the remaining contribution of the spin-spin correlation due to the unideal detection efficiency for the decays of the opposite tau should be studied with realistic MC simulation.
- *BABAR* and Belle performed complementary study of the radiative leptonic τ decay

  ($\tau \to \ell\nu\nu\gamma$ ($\ell = e,\ \mu$)):
  - With the statistics of 431 fb^{-1} branching fractions were measured with the relative accuracy better than 3% by *BABAR*:

    $$\mathcal{B}(\tau \to \mu\nu\nu\gamma)[E^*_\gamma > 10\,\mathrm{MeV}] = (3.69 \pm 0.03 \pm 0.10) \times 10^{-3}$$
    $$\mathcal{B}(\tau \to e\nu\nu\gamma)[E^*_\gamma > 10\,\mathrm{MeV}] = (1.847 \pm 0.015 \pm 0.052) \times 10^{-2}$$
  - For the first time Belle measured Michel parameters, $\bar{\eta}$ and $\xi\kappa$ in $\tau \to \ell\nu\nu\gamma$ decays on the statistics of 703 fb^{-1}:

    $$\bar{\eta} = -1.3 \pm 1.5 \pm 0.8$$
    $$\xi\kappa = 0.5 \pm 0.4 \pm 0.2$$

  An importance of the NLO corrections and doubly-radiative decays was realized for the precision measurement of the branching ratios.
- Good potential for the Super Charm-Tau factory to improve the results obtained at B factories and compete with Belle II.
- Five-body leptonic decays are studied at Belle.
- Good potential for the Super Charm-Tau factory to improve Belle results and compete with Belle II: discover $\tau \to e\mu\mu\nu\nu$ and $\tau \to \mu\mu\mu\nu\nu$ decays and measure Michel parameters.

Backup slides

Michel parameters

$$\rho = \frac{3}{4} - \frac{3}{4}\left(|g^V_{LR}|^2 + |g^V_{RL}|^2 + 2|g^T_{LR}|^2 + 2|g^T_{RL}|^2 + \Re(g^S_{LR}g^{T*}_{LR} + g^S_{RL}g^{T*}_{RL})\right)$$

$$\eta = \frac{1}{2}\Re\left(6g^V_{RL}g^{T*}_{LR} + 6g^V_{LR}g^{T*}_{RL} + g^S_{RR}g^{V*}_{LL} + g^S_{RL}g^{V*}_{LR} + g^S_{LR}g^{V*}_{RL} + g^S_{LL}g^{V*}_{RR}\right)$$

$$\xi = 4\Re(g^S_{LR}g^{T*}_{LR}) - 4\Re(g^S_{RL}g^{T*}_{RL}) + |g^V_{LL}|^2 + 3|g^V_{LR}|^2 - 3|g^V_{RL}|^2 - |g^V_{RR}|^2 +$$

$$+5|g^T_{LR}|^2 - 5|g^T_{RL}|^2 + \frac{1}{4}|g^S_{LL}|^2 - \frac{1}{4}|g^S_{LR}|^2 + \frac{1}{4}|g^S_{RL}|^2 - \frac{1}{4}|g^S_{RR}|^2$$

$$\xi\delta = \frac{3}{16}|g^S_{LL}|^2 - \frac{3}{16}|g^S_{LR}|^2 + \frac{3}{16}|g^S_{RL}|^2 - \frac{3}{16}|g^S_{RR}|^2 - \frac{3}{4}|g^T_{LR}|^2 + \frac{3}{4}|g^T_{RL}|^2 +$$

$$+\frac{3}{4}|g^V_{LL}|^2 - \frac{3}{4}|g^V_{RR}|^2 + \frac{3}{4}\Re(g^S_{LR}g^{T*}_{LR}) - \frac{3}{4}\Re(g^S_{RL}g^{T*}_{RL})$$

$$\bar{\eta} = \left|g^V_{RL}\right|^2 + \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 + \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 + \left|g^T_{LR}\right|^2\right)$$

$$\xi\kappa = \left|g^V_{RL}\right|^2 - \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 - \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 - \left|g^T_{LR}\right|^2\right)$$