Precision studies of leptonic $\tau$ decays

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3. Study of $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau \gamma$ decay
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5. Summary
The world largest statistics of $\tau$ leptons collected by $e^+ e^- B$ factories (Belle and \textit{BABAR}) opens new era in the precision tests of the Standard Model (SM).

Basic tau properties, like: lifetime, mass, couplings, electric dipole moment, anomalous magnetic dipole moment, etc. should be measured experimentally as precisely as possible in order to test SM and search for the effects of New Physics.

In the SM $\tau$ decays due to the charged weak interaction described by the exchange of $W^\pm$ with a pure vector coupling to only left-handed fermions. There are two main classes of tau decays:

- Decays with leptons, like: $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$, $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau \gamma$, $\tau^- \rightarrow \ell^- \ell'^+ \ell'^- \bar{\nu}_\ell \nu_\tau$; $\ell, \ell' = e, \mu$. They provide very clean laboratory to probe electroweak couplings, which is complementary/competitive to precision studies with muon (in experiments with muon beam). Plenty of New Physics models can be tested/constrained in the precision studies of the dynamics of decays with leptons.

- Hadronic decays of $\tau$ offer unique tools for the precision study of low energy QCD.
Integrated luminosity is 1.55 ab$^{-1}$

$\sigma(b\bar{b}) = 1.05 \text{ nb} \quad N_{b\bar{b}} = 1.2 \times 10^9$

$\sigma(c\bar{c}) = 1.30 \text{ nb} \quad N_{c\bar{c}} = 2.0 \times 10^9$

$\sigma(\tau\tau) = 0.92 \text{ nb} \quad N_{\tau\tau} = 1.4 \times 10^9$

$B$ factories are also charm and $\tau$ factories!
Planned integrated luminosity is 50 ab$^{-1}$

\[ \sigma(b\bar{b}) = 1.05 \text{ nb} \quad N_{b\bar{b}} = 53 \times 10^9 \]
\[ \sigma(c\bar{c}) = 1.30 \text{ nb} \quad N_{c\bar{c}} = 65 \times 10^9 \]
\[ \sigma(\tau\tau) = 0.92 \text{ nb} \quad N_{\tau\tau} = 46 \times 10^9 \]
In five c.m.s. energy points 
\( (2E = 3.554, 3.686, 3.770, 4.170, 4.650 \text{ GeV}) \) it is planned to 
accumulate \( 7 \text{ ab}^{-1} \), which corresponds to 
\( N_{\tau\tau} = 21 \times 10^9 \), which is 
2.2 times smaller than the planned \( \tau\tau \) statistics at Belle II. 
However, the crucial feature of the Super Charm-Tau Factory project, 
the **polarized electron beam** and **lower c.m.s. energies**, might give 
some advantages in \( \tau \) lepton studies in comparison with Belle II, thus, 
compensating smaller statistics of taus.
Precision studies of $\tau$ at $e^+e^-$ factories

Michel parameters in $\tau \to \ell \nu \nu (\rho, \eta, \xi, \delta)$:

**Belle**: Systematic uncertainties are about $(1 \div 3)\%$; arXiv:1409.4969

Study of the radiative leptonic decays $\tau \to \ell \nu \nu \gamma$:

**BABAR**: Measurement of $B(\tau \to \ell \nu \nu \gamma)$; PRD 91, 051103(R) (2015)

**Belle**: $\bar{\eta} = -1.3 \pm 1.5 \pm 0.8$, $\xi \kappa = 0.5 \pm 0.4 \pm 0.2$; arXiv:1709.08833

Study of the 5-lepton decays $\tau \to \ell\ell'\ell'\nu\nu$:

**CLEO**: $B(\tau \to eee\nu\nu) = (2.8 \pm 1.5) \times 10^{-5}$,

$B(\tau \to \mu e e \nu \nu) < 3.6 \times 10^{-5} \text{ (CL } = 90\%)$; PRL 76, 2637 (1996)

**Belle**: statistical uncertainties are about $(3 \div 5)\%$; J. Phys. Conf. Ser. 912 (2017) no.1, 012002.

Lepton universality with $\tau \to \ell \nu \nu$ and $\tau \to h \nu$ ($h=\pi, K$):

**BABAR**: $\left( \frac{g_\mu}{g_e} \right)_\tau = 1.0036 \pm 0.0020$, $\left( \frac{g_\tau}{g_\mu} \right)_h = 0.9850 \pm 0.0054$; PRL 105, 051602 (2010)

Tau lifetime:

**Belle**: $\tau_\tau = (290.17 \pm 0.53\text{(stat)} \pm 0.33\text{(syst)}) \text{ fs}$; PRL 112, 031801 (2014)

**BABAR** (prelim.): $\tau_\tau = (289.40 \pm 0.91\text{(stat)} \pm 0.90\text{(syst)}) \text{ fs}$; Nucl. Phys. B 144, 105 (2005)

Tau mass:

**BES3**: $m_\tau = (1776.91 \pm 0.12\text{(stat)} \pm 0.10\text{(syst)}) \text{ MeV}/c^2$; PRD 90, 012001 (2014)

**KEDR**: $m_\tau = (1776.81 \pm 0.25\text{ (stat)} \pm 0.15\text{(syst)}) \text{ MeV}/c^2$; JETPL 85, 347 (2007)

**Belle**: $m_\tau = (1776.61 \pm 0.13\text{(stat)} \pm 0.35\text{(syst)}) \text{ MeV}/c^2$; PRL 99, 011801 (2007)

**BABAR**: $m_\tau = (1776.68 \pm 0.12\text{(stat)} \pm 0.41\text{(syst)}) \text{ MeV}/c^2$; PRD 80, 092005 (2009)
Michel parameters

In the SM charged weak interaction is described by the exchange of $W^\pm$ with a pure vector coupling to only left-handed fermions ("V-A" Lorentz structure). Deviations from "V-A" indicate New Physics. $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau$ ($\ell = e, \mu$) decays provide clean laboratory to probe electroweak couplings.

The most general, Lorentz invariant four-lepton interaction matrix element:

$$\mathcal{M} = \frac{4G}{\sqrt{2}} \sum_{N=S,V,T} \sum_{i,j=L,R} g^N_{ij} \left[ \bar{u}_i(l^-) \Gamma^N \nu_n(\bar{\nu}_l) \right] \left[ \bar{u}_m(\nu_\tau) \Gamma_N u_j(\tau^-) \right],$$

$$\Gamma^S = 1, \quad \Gamma^V = \gamma^\mu, \quad \Gamma^T = \frac{i}{2\sqrt{2}}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu)$$

Ten couplings $g^N_{ij}$, in the SM the only non-zero constant is $g^V_{LL} = 1$

Four bilinear combinations of $g^N_{ij}$, which are called as Michel parameters (MP): $\rho$, $\eta$, $\xi$ and $\delta$ appear in the energy spectrum of the outgoing lepton:

$$\frac{d\Gamma(\tau^\pm)}{d\Omega dx} = \frac{4G^2 M_\tau E_{\text{max}}^4}{(2\pi)^4} \sqrt{x^2 - x_0^2} \left( x(1-x) + \frac{2}{9} \rho (4x^2 - 3x - x_0^2) + \eta x_0 (1-x) \right)$$

$$\mp \frac{1}{3} P_\tau \cos \theta_\ell \xi \sqrt{x^2 - x_0^2} \left[ 1 - x + \frac{2}{3} \delta (4x - 4 + \sqrt{1 - x_0^2}) \right], \quad x = \frac{E_\ell}{E_{\text{max}}}, \quad x_0 = \frac{m_\ell}{E_{\text{max}}}$$

In the SM: $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$
## Status of Michel parameters in $\tau$ decays

<table>
<thead>
<tr>
<th>Michel par.</th>
<th>Measured value</th>
<th>Experiment</th>
<th>SM value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$0.747 \pm 0.010 \pm 0.006$</td>
<td>CLEO-97 3/4</td>
<td>$1.2%$</td>
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<tr>
<td>(e or $\mu$)</td>
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<tr>
<td>$\eta$</td>
<td>$0.012 \pm 0.026 \pm 0.004$</td>
<td>ALEPH-01 0</td>
<td>$2.6%$</td>
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<tr>
<td>$\xi$</td>
<td>$1.007 \pm 0.040 \pm 0.015$</td>
<td>CLEO-97 1</td>
<td>$4.3%$</td>
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<tr>
<td>(e or $\mu$)</td>
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<tr>
<td>$\xi_\delta$</td>
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<tr>
<td>$\xi_\text{h}$</td>
<td>$0.992 \pm 0.007 \pm 0.008$</td>
<td>ALEPH-01 1</td>
<td>$1.1%$</td>
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<tr>
<td>(all hadr.)</td>
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</table>
Status of Michel parameters in $\tau$ decays

With Belle statistics, which is about 300 times larger than the previous experimental $\tau\tau$ data samples, we can improve MP uncertainties by one order of magnitude.

In BSM models the couplings to $\tau$ are expected to be larger than those to $\mu$. Contribution from New Physics in $\tau$ decays can be enhanced by a factor of $(\frac{m_\tau}{m_\mu})^2$.

- **Type II 2HDM:**
  \[ \eta_\mu(\tau) = \frac{m_\mu M_\tau}{2} \left( \frac{\tan^2 \beta}{M_{H^\pm}^2} \right)^2; \quad \frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500 \]

- **Tensor interaction:**
  \[ \mathcal{L} = \frac{g}{2\sqrt{2}} W^\mu \left\{ \bar{\nu} \gamma_\mu (1 - \gamma^5) \tau + \frac{\kappa_W}{2m_\tau} \partial^\nu \left( \bar{\nu} \sigma_\mu \nu (1 - \gamma^5) \tau \right) \right\}, \]
  \[-0.096 < \kappa_W < 0.037: \text{DELPHI} \text{ Abreu EPJ C16 (2000) 229.} \]

- **Lorentz and CPTV:** Hollenberg PLB 701 (2011) 89
- **$\mu - \tau$ LFV Yukawa couplings in $\xi_\mu$:** K. Tobe, JHEP 1610 (2016) 114
Spin-dependent measurements with $\tau$

To measure $\xi$ and $\delta$ MP we have to know $\tau$ spin direction. At B factories, the effect of $\tau$ spin-spin correlation in $e^+e^- \rightarrow \tau^+(\vec{\zeta}^+)\tau^- (\vec{\zeta}^-)$ can be used.

At the Super Charm-Tau factory with polarized electron beam the average polarization of single $\tau$ is nonzero, hence the differential decay probability will contain both, $\tau$ spin-dependent and spin-independent parts.

$$
\frac{d\sigma(\vec{\zeta}^-, \vec{\zeta}^+)}{d\Omega_\tau} = \frac{\alpha^2}{64E^2_\tau} \beta_\tau (D_0 + D_{ij}\zeta_i^- \zeta_j^+ + \mathcal{P}_e(F_i^- \zeta_i^- + F_j^+ \zeta_j^+))
$$

$$
D_0 = 1 + \cos^2 \theta + \frac{1}{\gamma^2_\tau} \sin^2 \theta, \quad \mathcal{P}_e = \frac{N_e(+) - N_e(-)}{N_e(+) + N_e(-)}
$$

$$
D_{ij} = \begin{pmatrix}
(1 + \frac{1}{\gamma^2_\tau}) \sin^2 \theta & 0 & \frac{1}{\gamma_\tau} \sin 2\theta \\
0 & -\beta^2_\tau \sin^2 \theta & 0 \\
\frac{1}{\gamma_\tau} \sin 2\theta & 0 & 1 + \cos^2 \theta - \frac{1}{\gamma^2_\tau} \sin^2 \theta
\end{pmatrix}
$$

Single $\tau$ studies at the Super Charm-Tau factory:

$$
\frac{d\sigma(\vec{\zeta}^-)}{d\Omega_\tau} = \frac{\alpha^2}{32E^2_\tau} \beta_\tau (D_0 + \mathcal{P}_e F_i^- \zeta_i^-)
$$
At B factory: study of $(\ell\nu\nu; \rho\nu)$ and $(\rho\nu; \rho\nu)$ events

Effect of $\tau$ spin-spin correlation is used to measure $\xi$ and $\delta$ MP.

Events of the $(\tau^\mp \to \ell^\mp \nu\nu; \tau^\pm \to \rho^\pm \nu)$ topology are used to measure: $\rho$, $\eta$, $\xi_\rho \xi$ and $\xi_\rho \xi \delta$, while $(\tau^\mp \to \rho^\mp \nu; \tau^\pm \to \rho^\pm \nu)$ events are used to extract $\xi_\rho^2$.

$$
\frac{d\sigma(\ell^\mp \nu\nu, \rho^\pm \nu)}{dE^*_\ell d\Omega^*_\ell d\Omega^*_\rho d\bar{\Omega}_\pi d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho \xi A_3 + \xi_\rho \xi \delta A_4 = \sum_{i=0}^4 A_i \Theta_i 
$$

$$
\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp \nu\nu, \rho^\pm \nu)}{dp_\ell d\Omega_\ell dp_\rho d\Omega_\rho d\bar{\Omega}_\pi d\Omega_\tau} = \int_{\Phi_1} \Phi_2 \frac{d\sigma(\ell^\mp \nu\nu, \rho^\pm \nu)}{dE^*_\ell d\Omega^*_\ell d\Omega^*_\rho d\bar{\Omega}_\pi d\Omega_\tau} \left| \frac{\partial(E^*_\ell, \Omega^*_\ell, \Omega^*_\rho, \Omega_\tau, \Phi_\tau)}{\partial(p_\ell, \Omega_\ell, p_\rho, \Omega_\rho, \Phi_\tau)} \right| d\Phi_\tau 
$$

$$
L = \prod_{k=1}^N \mathcal{P}^{(k)}, \quad \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/N(\tilde{\Theta}), \quad N(\tilde{\Theta}) = \int \mathcal{F}(\vec{z}) d\vec{z}, \quad \tilde{\Theta} = (1, \rho, \eta, \xi_\rho \xi_\ell, \xi_\rho \xi_\ell \delta_\ell) 
$$

$$
\mathcal{P}_{\text{total}} = (1 - \sum_{i=1}^4 \lambda_i) \mathcal{P}_{\text{signal}}^\rho - \rho + \lambda_1 \mathcal{P}_{bg}^{\rho - \rho} + \lambda_2 \mathcal{P}_{bg}^{\rho - \rho} + \lambda_3 \mathcal{P}_{bg}^{\rho - \rho} + \lambda_4 \mathcal{P}_{bg}^{\rho - \rho} \text{ (MC)} 
$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu; \rho\nu)$ events in the 9D phase space $\vec{z} = (p_\ell, \cos \theta_\ell, \phi_\ell, p_\rho, \cos \theta_\rho, \phi_\rho, m^2_{\pi\pi}, \cos \tilde{\theta}_\pi, \tilde{\phi}_\pi)$ in CMS.
Method, $\tau^- \rightarrow h^- \nu_\tau$, $h = \pi, \rho$

\[ J^\mu = \langle h | \bar{d} \gamma^\mu (c_V + c_A \gamma^5) u | 0 \rangle \]

Michel formalism for the $\tau^- \rightarrow h^- \nu_\tau$ includes:

\[ \xi_h = -\frac{2 \text{Re}(c_V^* c_A)}{|c_V|^2 + |c_A|^2} = -h_{\nu_\tau} \quad (\text{=1 in SM}) \]

\[ \frac{d\Gamma(\tau^\mp \rightarrow \pi^\mp \nu)}{d\Omega_\pi} = C(1 \pm \xi_\pi P_\tau \cos \theta_\pi) \]

\[ \frac{d\Gamma(\tau^\mp \rightarrow \rho^\mp \nu)}{dm^2_{\pi\pi} d\Omega_\rho d\Omega^*_{\pi}} = f(\vec{k}_1, \vec{k}_2) \pm \xi_\rho \bar{P}_\tau \vec{g}(\vec{k}_1, \vec{k}_2) = f(\vec{k}_1, \vec{k}_2)(1 \pm \xi_\rho \bar{P}_\tau \vec{H}_\rho) \]

\[ \vec{H}_\rho = M_\tau \frac{2(q, Q)\vec{Q} + Q^2 \vec{K}}{2(p, Q)(q, Q) - Q^2(p, q)} \text{-polarimeter vector} \]

Precision measurement of $\tau$ neutrino helisity, $h_{\nu_\tau}$, in various decay modes is an important test of the Standard Model.

Helicity sensitive variable $\omega$ is introduced as:

$$\omega = \frac{1}{\Phi_2 - \Phi_1} \int_{\Phi_1}^{\Phi_2} (\vec{H}_\rho \pm, \vec{n}_\tau \pm) d\Phi = \langle (\vec{H}_\rho \pm, \vec{n}_\tau \pm) \rangle_{\Phi_\tau}$$

Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.
Method, theoretical framework

  \( \ell_1^\pm - \ell_2^\pm, \ell^\pm - h^\pm, \ell = e, \mu; \ h = \pi, \ K. \)

  \( \ell^\pm - \rho^\pm (\rightarrow \pi^\pm \pi^0) + \text{feasibility study.} \)

\[ \frac{d\sigma(\zeta', \zeta')}{d\Omega} = \frac{\alpha^2}{64E^2_{\tau}} \beta_{\tau}(D_0 + D_{ij}\zeta_i \zeta_j') \]

\[ \frac{d\Gamma(\tau^\pm(\zeta^*) \rightarrow \ell^\pm \nu \nu)}{dx^* d\Omega^*_\ell} = \kappa_{\ell} (A(x^*) \mp \xi_{\ell} \bar{n}_{\ell} \zeta^* B(x^*)) , \ x^* = E^*_\ell / E^*_{\ell \max} \]

\[ A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*) , \ B(x^*) = B_1(x^*) + \delta B_2(x^*) \]

\[ \frac{d\Gamma(\tau^\pm(\zeta^*) \rightarrow \rho^\pm \nu)}{dm^2_{\pi \pi} d\Omega^*_{\rho} d\bar{\Omega}_{\pi}} = \kappa_{\rho} (A' \mp \xi_{\rho} \bar{B'} \zeta^* \zeta^*) W(m^2_{\pi \pi}) \]

\[ A' = 2(q, Q) Q_0^* - Q^2 q_0^* , \ B' = Q^2 \bar{K}^* + 2(q, Q) \bar{Q}^* , \ W = |F_{\pi}(m^2_{\pi \pi})|^2 \frac{p_{\rho}(m^2_{\pi \pi}) \bar{p}_{\pi}(m^2_{\pi \pi})}{M_{\tau} m_{\pi \pi}} \]

\[ \frac{d\sigma(\ell^\pm, \rho^\pm)}{dE^*_{\ell} d\Omega^*_\rho d\Omega^*_\rho dm^2_{\pi \pi} d\bar{\Omega}_{\pi} d\Omega_{\tau}} = \kappa_{\ell} \kappa_{\rho} \frac{\alpha^2 \beta_{\tau}}{64E^2_{\tau}} (D_0 A' A(E^*_{\ell}) + \xi_{\rho} \xi_{\ell} D_{ij} n_{\ell i}^* B_j B(E^*_{\ell})) W(m^2_{\pi \pi}) \]

\[ \frac{d\sigma(\ell^\pm, \rho^\pm)}{dp_{\ell} d\Omega_{\ell} dp_{\rho} d\Omega_{\rho} dm^2_{\pi \pi} d\bar{\Omega}_{\pi}} = \int_{\Phi_1} \frac{d\sigma(\ell^\pm, \rho^\pm)}{dE^*_{\ell} d\Omega^*_\ell d\Omega^*_\rho dm^2_{\pi \pi} d\bar{\Omega}_{\pi} d\Omega_{\tau}} \left| \frac{\partial(E^*_{\ell}, \Omega^*_{\ell}, \Omega^*_{\rho}, \Omega_{\tau})}{\partial(p_{\ell}, \Omega_{\ell}, p_{\rho}, \Omega_{\rho}, \Phi_{\tau})} \right| d\Phi_{\tau} \]
4 Michel parameters \( \vec{\Theta} = (1, \rho, \eta, \xi_\rho, \xi_\ell, \xi_\rho \xi_\ell \delta_\ell) \) are extracted in the unbinned maximum likelihood fit of \( (\ell \nu \nu; \rho \nu) \) events in the 9D phase space in CMS, \( \vec{z} = (p_\ell, \cos \theta_\ell, \phi_\ell, p_\rho, \cos \theta_\rho, \phi_\rho, m_{\pi \pi}, \cos \tilde{\theta}_\pi, \tilde{\phi}_\pi) \). The PDF for individual k-th event is written in the form:

\[
P^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})}, \quad \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z}) d\vec{z}
\]

Likelihood function for N events:

\[
L = \prod_{k=1}^{N} P^{(k)}, \quad \mathcal{L} = -\ln L = N \ln \mathcal{N}(\vec{\Theta}) - \sum_{k=1}^{N} \ln \mathcal{F}^{(k)}, \quad \mathcal{F}^{(k)} = \mathcal{F}(\vec{z}^{(k)})
\]

\[
\mathcal{F}^{(k)} = A_0^{(k)} \Theta_0 + A_1^{(k)} \Theta_1 + A_2^{(k)} \Theta_2 + A_3^{(k)} \Theta_3 + A_4^{(k)} \Theta_4 = \sum_{i=0}^{4} A_i^{(k)} \Theta_i
\]

\[
\mathcal{N} = C_0 \Theta_0 + C_1 \Theta_1 + C_2 \Theta_2 + C_3 \Theta_3 + C_4 \Theta_4, \quad C_j = \frac{1}{N} \sum_{k=1}^{N} C_j^{(k)}, \quad C_j^{(k)} = \frac{A_j^{(k)}}{\sum_{i=0}^{4} A_i^{(k)} \Theta_i^{MC}}
\]

\[
\vec{\Theta}^{MC} = (1, 0.75, 0, 1, 0.75), \quad \mathcal{L} = N \ln \left( \sum_{j=0}^{4} C_j \Theta_j \right) - \sum_{k=1}^{N} \ln \left( \sum_{i=0}^{4} A_i^{(k)} \Theta_i \right)
\]

As a result fitted statistics is represented by a set of \( 5 \times N \) values of \( A_i^{(k)} \) \( (k = 1 \div N, \ i = 0 \div 4) \), which is calculated only once.

\( C_i \ (i = 0 \div 4) \) are calculated using MC simulation.

In ideal case (no rad. corr., \( \varepsilon = 100\% \)): \( C_0 = 1, \ C_2 = 4m_\ell / m_\tau, \ C_{1,3,4} = 0 \)
Suppose we have $N_{MC}$ MC events, which were simulated with particular set $\vec{\Theta}^{MC}$. By reweighting each event we can calculate normalization for arbitrary set $\vec{\Theta}$:

$$N(\vec{\Theta}) \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} w^{(k)}, \quad w^{(k)} = \frac{A_i^{(k)} \Theta_i}{A_j^{(k)} \Theta_j^{MC}} = B_m^{(k)} \Theta_m, \quad B_m^{(k)} = \frac{A_m^{(k)}}{A_j^{(k)} \Theta_j^{MC}}$$

$$N(\vec{\Theta}) = C_i \Theta_i, \quad C_i = \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} B_i^{(k)}$$

This algorithm can be easily extended to take into account selection efficiency:

$$\mathcal{F}(\vec{z}) \rightarrow \mathcal{F}'(\vec{z}) = \mathcal{F}(\vec{z}) \epsilon(\vec{z}), \quad \mathcal{N}'(\vec{\Theta}) = \int \mathcal{F}(\vec{z}) \epsilon(\vec{z}) d\vec{z}$$

$$\mathcal{L} = N_{sel} \ln \mathcal{N}'(\vec{\Theta}) - \sum_{k=1}^{N_{sel}} \ln(\mathcal{F}^{(k)} \epsilon(\vec{z})) = N_{sel} \ln(C'_i \Theta_i) - \sum_{k=1}^{N_{sel}} \ln(A_i^{(k)} \Theta_i) - \sum_{k=1}^{N_{sel}} \ln(\epsilon(\vec{z}))$$

$$C'_i = \frac{1}{N_{MC}} \sum_{k=1}^{N_{sel}} B_i^{(k)}$$

Accuracy of the evaluation of the $C'_i$ coefficients is crucial in the precision measurement of Michel parameters.
Corrections, detector effects, background

**Physical corrections:**
- All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+ e^- \rightarrow \tau^+ \tau^- (\gamma)$ are included
- Radiative leptonic decays $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau \gamma$
- Radiative decay $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau \gamma$

**Detector effects:**
- Track momentum resolution
- $\gamma$ energy and angular resolution
- Effect of external bremsstrahlung for $e^- \rho$ events
- Beam energy spread
- EXP/MC efficiency corrections (trigger, track rec., $\pi^0$ rec., $\ell$ID, $\pi$ID)

**Background:**
The main background comes from $(\ell \nu \nu; \pi 2 \pi^0 \nu)(\sim 10\%), (\pi \nu; \pi \pi^0 \nu)(\sim 1.5\%)$ and $(\rho^+ \nu; \rho^- \nu)(\sim 0.5\%)$ events, it is included in PDF analytically. The remaining background($\sim 2.0\%)$ is taken into account using MC-based approach.
Background from the non-$\tau \tau$ events is $\lesssim 0.1\%$. 
Physical corrections

Radiative corrections to $e^+ e^- \rightarrow \tau^+ \tau^-$
- All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+ e^- \rightarrow \tau^+ \tau^- (\gamma)$ are included:
- KKMC based approach:
  - We generate table of ISR photons and then use it to calculate visible differential cross section in CMS.

Radiative leptonic decays $\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau \gamma$
- Analytical approach based on:
  - A. Arbuzov, A. Czarnecki and A. Gaponenko, Phys. Rev. D 65 (2002) 113006. $\mathcal{O}(\alpha^2 \ln^2(\frac{m_\mu}{m_e}))$.
- TAUOLA based approach:

Radiative corrections to $\tau^- \rightarrow \pi^- \pi^0 \nu_\tau$
- Analytical approach based on:
- PHOTOS based approach
\( \mathcal{O}(\alpha^3) \) corrections to \( e^+ e^- \rightarrow \tau^+ \tau^- (\gamma) \)

Charge-odd part of the cross section comes from the interference of the ISR and FSR diagrams as well as box and Born diagrams, and \( Z^0 \)-exchange and Born diagrams.

Initial state radiation (ISR)

\[ \frac{d\sigma_{\text{vis}}(s)}{dp_{\ell} d\Omega_{\ell} dp_{\rho} d\Omega_{\rho} dm_{\pi} d\tilde{\Omega}_{\pi}} = \int_{0}^{1} dx_{1}dx_{2}D(x_{1})D(x_{2}) \frac{d\sigma(s(1-x_{1})(1-x_{2}))}{dp_{\ell} d\Omega_{\ell} dp_{\rho} d\Omega_{\rho} dm_{\pi} d\tilde{\Omega}_{\pi}} \left| \frac{\partial(p'_{\ell}, \Omega'_{\ell})}{\partial(p_{\ell}, \Omega_{\ell})} \right| \left| \frac{\partial(p'_{\rho}, \Omega'_{\rho})}{\partial(p_{\rho}, \Omega_{\rho})} \right| \]

- \( D(x) = x^{\beta/2-1} h(x) \) - probability function for initial \( e^{\pm} \) to emit a \( \gamma \)-quantum jet carrying \( x_{1,2} \) part of \( e^{\pm} \) energy \( E_{\text{beam}} = \sqrt{s}/2 \). \( \beta = \frac{2\alpha}{\pi} (\ln \frac{s}{m^{2}} - 1) \), \( h(x) \) - smooth limited function.

- \( \left| \frac{\partial(p'_{i}, \Omega'_{i})}{\partial(p_{i}, \Omega_{i})} \right| \) \( (i = \ell, \rho) \) - Jacobian of transformation from the \( \tau^{+}\tau^{-} \) rest frame to the Belle CMS.

At the Super Charm-Tau factory the impact of the ISR is expected to be essentially smaller.
\[ p(x) = \frac{\varepsilon(x)}{\bar{\varepsilon}} \left( 1 - \sum_i \lambda_i \right) \frac{S(x)}{\int \frac{\varepsilon(x)}{\bar{\varepsilon}} S(x) dx} + \lambda_{3\pi} \frac{\tilde{B}_{3\pi}(x)}{\int \frac{\varepsilon(x)}{\bar{\varepsilon}} \tilde{B}_{3\pi}(x) dx} + \lambda_{\pi} \frac{\tilde{B}_{\pi}(x)}{\int \frac{\varepsilon(x)}{\bar{\varepsilon}} \tilde{B}_{\pi}(x) dx} + \lambda_{\rho} \frac{\tilde{B}_{\rho}(x)}{\int \frac{\varepsilon(x)}{\bar{\varepsilon}} \tilde{B}_{\rho}(x) dx} + \left( 1 - \sum_i \lambda_i \right) \frac{N_{\text{rest}}^{\text{sel}}(x)}{N_{\text{sig}}^{\text{sel}}(x)} S_{\text{SM}}(x) \]

\[ \tilde{B}_{3\pi}(x) = \int 2(1 - \varepsilon_{\pi^0}(y)) \varepsilon_{\text{add}}(y) B_{3\pi}(x, y) dy, \quad \tilde{B}_{\pi}(x) = \frac{\varepsilon_{\mu \rightarrow \mu|\pi}(p_\ell, \Omega_\ell)}{\varepsilon_{\mu \rightarrow \mu|\mu}(p_\ell, \Omega_\ell)} B_{\pi}(x), \quad \tilde{B}_{\rho}(x) = \frac{\varepsilon_{\text{ID}\pi \rightarrow \mu}(p_\ell, \Omega_\ell)}{\varepsilon_{\text{ID}\mu \rightarrow \mu}(p_\ell, \Omega_\ell)} \int \frac{1 - \varepsilon_{\pi^0}(y)}{\varepsilon_{\text{add}}(y) B_{\rho}(x, y) dy, \quad \overline{\varepsilon(x)} = \varepsilon_{\text{corr}}(x) \varepsilon(x) \]

- \( x = (p_\ell, \Omega_\ell, p_\rho, \Omega_\rho, m_{\pi\pi}^2, \tilde{\Omega}_{\pi}) \); \( y = (p_{\pi^0}, \Omega_{\pi^0}) \);
- \( S(x) \) - theoretical density of signal \((\ell^+\nu\nu, \rho^\pm\nu)\) events;
- \( B_{3\pi}(x, y) \) - theoretical density of background \((\ell^+\nu\nu, \pi^\pm2\pi^0\nu)\) events;
- \( B_{\pi}(x) \) - theoretical density of background \((\pi^\pm\nu, \rho^\pm\nu)\) events;
- \( B_{\rho}(x) \) - theoretical density of background \((\rho^\pm\nu, \rho^\pm\nu)\) events;
- \( \varepsilon(x) \) - detection efficiency for signal events (common multiplier);
- \( N_{\text{rest}}^{\text{sel}}(x)/N_{\text{sig}}^{\text{sel}}(x) \) - number of the selected (remaining/signal) MC events in the multidimensional cell around "x". Admixture of the remaining background is \((1 \div 2\%)\).
- \( \lambda_i \) - i-th background fraction (from MC);
- \( \varepsilon_{\pi^0}(y) \) - \( \pi^0 \) detection efficiency (tabulated from MC);
- \( \varepsilon_{\text{add}}(y) = \varepsilon_{3\pi\text{add}}(y)/\varepsilon_{\text{sig}3\pi\text{add}} \) - ratio of the \( E_{\gamma\text{rest}}^{\text{LAB}} \) cut efficiencies (tabulated from MC);
- \( \varepsilon_{\mu \rightarrow \mu|\pi}(p_\ell, \Omega_\ell) / \varepsilon_{\mu \rightarrow \mu|\mu}(p_\ell, \Omega_\ell) \) is tabulated from MC;
- \( \varepsilon_{\text{corr}}(x) \) - EXP/MC efficiency correction.
### Systematic uncertainties

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta(\rho)$, %</th>
<th>$\Delta(\eta)$, %</th>
<th>$\Delta(\xi_\rho\xi)$, %</th>
<th>$\Delta(\xi_\rho\xi\delta)$, %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Physical corrections</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ISR+$\mathcal{O}(\alpha^3)$</td>
<td>0.10</td>
<td>0.30</td>
<td>0.20</td>
<td>0.15</td>
</tr>
<tr>
<td>$\tau \rightarrow \ell\nu\nu\gamma$</td>
<td>0.03</td>
<td>0.10</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>$\tau \rightarrow \rho\nu\gamma$</td>
<td>0.06</td>
<td>0.16</td>
<td>0.11</td>
<td>0.02</td>
</tr>
<tr>
<td>Background</td>
<td>0.20</td>
<td>0.60</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>Apparatus corrections</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Resolution $\oplus$ brems.</td>
<td>0.10</td>
<td>0.33</td>
<td>0.11</td>
<td>0.19</td>
</tr>
<tr>
<td>$\sigma(E_{\text{beam}})$</td>
<td>0.07</td>
<td>0.25</td>
<td>0.03</td>
<td>0.15</td>
</tr>
<tr>
<td><strong>Normalization</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta N$</td>
<td>0.11</td>
<td>0.50</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td><strong>without EXP/MC corr.</strong></td>
<td>0.29</td>
<td>0.95</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>$R_{\text{trg}}$</td>
<td>$\sim 1$</td>
<td>$\sim 2$</td>
<td>$\sim 3$</td>
<td>$\sim 3$</td>
</tr>
</tbody>
</table>
Super Charm-Tau factory, \( \tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau \)

\[
\frac{d\sigma(\vec{\zeta})}{d\Omega_\tau} = \frac{\alpha^2}{32E^2_\tau} \beta_\tau (D_0 + \mathcal{P}_e F_i \vec{\zeta}_i)
\]

\[
\frac{d\Gamma(\tau^\pm(\vec{\zeta}^*) \rightarrow \ell^\mp \nu\nu)}{dx^* d\Omega^*_\ell} = \kappa_\ell (A(x^*) \mp \xi_\ell \bar{n}_\ell \vec{\zeta}^* B(x^*), \ x^* = E^*_\ell / E^*_{\ell\text{max}}
\]

\[
A(x^*) = A_0(x^*) + \rho A_1(x^*) + \eta A_2(x^*), \ B(x^*) = B_1(x^*) + \delta B_2(x^*)
\]

\[
\frac{d\sigma(\ell^\mp)}{dE^*_\ell d\Omega^*_\ell d\Omega_\tau} = \kappa_\ell \frac{\alpha^2 \beta_\tau}{32E^2_\tau} (D_0 A(E^*_\ell) \mp \mathcal{P}_e \xi_\ell \bar{n}_\ell B(E^*_\ell))
\]

\[
\frac{d\sigma(\ell^\mp)}{dp_\ell d\Omega_\ell} = \int_{\Omega^- \text{-sector}} \frac{d\sigma(\ell^\mp)}{dE^*_\ell d\Omega^*_\ell d\Omega_\tau} \left| \frac{\partial(E^*_\ell, \Omega^*_\ell)}{\partial(p_\ell, \Omega_\ell)} \right| d\Omega_\tau
\]

\(\Omega^-\)-sector is determined by the kinematical constraint \(m_{\nu\nu} > 0\)

All Michel parameters \((\rho, \eta, \mathcal{P}_e \xi, \mathcal{P}_e \xi \delta)\) are measured in the unbinned maximum likelihood fit of \((\tau^- \rightarrow \ell^- \bar{\nu}_\ell \nu_\tau; \ \tau^+ \rightarrow \text{all})\) events in the 3D phase space. Due to the unideal detection efficiency for the decays of the opposite tau, there is still some contribution from the spin-spin correlation term.

The reduced 3D phase space allows one to tabulate various EXP/MC corrections to the detection efficiency more precisely.
Super Charm-Tau factory, $\tau^- \rightarrow \pi^- / \rho^- \nu_{\tau}$

\[
\frac{d\sigma(\zeta)}{d\Omega_{\tau}} = \frac{\alpha^2}{32E_{\tau}^2} \beta_{\tau} (D_0 + \mathcal{P}_e F_i \zeta_i) \\

\frac{d\Gamma(\tau^+ \rightarrow \pi^+ \nu)}{d\Omega_{\pi}^*} = \kappa_{\pi} (1 \pm \xi_{\pi} \zeta \eta_{\pi}^*), \quad \frac{d\Gamma(\tau^+ \rightarrow \rho^+ \nu)}{dm_{\pi\pi}^2 d\Omega_{\rho}^* \Omega_{\pi}} = f(\vec{k}_1, \vec{k}_2) (1 \pm \xi_{\rho} \zeta \eta_{\rho}^*) \\

\frac{d\sigma(\pi^+)}{d\Omega_{\pi}^* d\Omega_{\tau}} = \kappa_{\pi} \frac{\alpha^2 \beta_{\tau}}{32E_{\tau}^2} (D_0 \pm \mathcal{P}_e \xi_{\pi} F_i \eta_{\pi i}) \\

\frac{d\sigma(\rho^+)}{d\Omega_{\rho}^* dm_{\pi\pi}^2 \Omega_{\pi} d\Omega_{\tau}} = f(\vec{k}_1, \vec{k}_2) \frac{\alpha^2 \beta_{\tau}}{32E_{\tau}^2} (D_0 \pm \mathcal{P}_e \xi_{\rho} F_i \eta_{\rho i}) \\

\frac{d\sigma(\pi^+)}{dp_{\pi} d\Omega_{\pi}} = \int_0^{2\pi} \frac{d\sigma(\pi^+)}{d\Omega_{\pi}^* d\Omega_{\tau}} \left| \frac{\partial(\Omega_{\pi}^*, \Omega_{\tau})}{\partial(p_{\pi}, \Omega_{\pi}, \Phi_{\tau})} \right| d\Phi_{\tau} \\

\frac{d\sigma(\rho^+)}{dp_{\rho} d\Omega_{\rho} dm_{\pi\pi}^2 \Omega_{\pi}} = \int_0^{2\pi} \frac{d\sigma(\rho^+)}{d\Omega_{\rho}^* dm_{\pi\pi}^2 \Omega_{\pi} d\Omega_{\tau}} \left| \frac{\partial(\Omega_{\rho}^*, \Omega_{\tau})}{\partial(p_{\rho}, \Omega_{\rho}, \Phi_{\tau})} \right| d\Phi_{\tau}
\]

Parameters ($\mathcal{P}_e \xi_{\pi}, \mathcal{P}_e \xi_{\rho}$) are measured in the unbinned maximum likelihood fit of the ($\tau^- \rightarrow \pi^- / \rho^- \nu_{\tau}; \tau^+ \rightarrow \text{all}$) events. These decays can be used to monitor $\mathcal{P}_e$ with high precision.
Photon carries information about spin state of outgoing lepton, as a result two additional parameters, \( \bar{\eta} \) and \( \xi \kappa \), can be extracted. These parameters were measured in \( \tau \) decays at Belle for the first time.
Michel parameters in $\tau \rightarrow \ell \nu\nu\gamma$, ($\ell = e, \mu$) (II)

$N_{\tau\tau} = 646 \times 10^6$, selected: 71171 ($\mu\nu\nu\gamma; \rho\nu$) and 77634 ($e\nu\nu\gamma; \rho\nu$) events

$\eta = -1.3 \pm 1.5 \pm 0.8$

$\xi \kappa = 0.5 \pm 0.4 \pm 0.2$
Measurement of $\mathcal{B}(\tau \to \ell \nu \nu \gamma)$ at BABAR (I)

$\int L dt = 431 \text{ fb}^{-1}$

Selections:

- 2-track events with zero net charge and 1 photon with $E_\gamma > 50$ MeV;
- $0.9 < \text{thrust} < 0.995$, signal hemisphere: $\ell + \gamma$, tag hemisphere: track+neutrals;
- reject $\ell^+ - \ell^-$ events, $E_{\text{tot}} < 9$ GeV, distance between track and photon clusters $d_{\ell \gamma} < 100$ cm.

- $\nu \nu \gamma$: $0.22 \leq E_\gamma \leq 2.0$ GeV, $M_{e\gamma} \geq 0.14$ GeV/c$^2$, $\cos \theta_{e\gamma} \geq 0.97$, $8 \leq d_{e\gamma} \leq 65$ cm
- $\mu \nu \nu \gamma$: $0.10 \leq E_\gamma \leq 2.5$ GeV, $M_{\mu\gamma} \leq 0.25$ GeV/c$^2$, $\cos \theta_{\mu\gamma} \geq 0.99$, $6 \leq d_{\mu\gamma} \leq 30$ cm

$$N_{\text{sel}}(\mu \nu \nu \gamma) = 15688 \pm 125 \quad N_{\text{sel}}(\nu \nu \gamma) = 18149 \pm 135$$
Measurement of \( \mathcal{B}(\tau \to \ell \nu \nu \gamma) \) at BABAR (II)

\[
\mathcal{B} = \frac{N_{\text{sel}}(1 - f_{\text{bg}})}{2\sigma_{\tau\tau} \mathcal{L} \varepsilon}
\]

<table>
<thead>
<tr>
<th>( \mu \nu \nu \gamma )</th>
<th>( \ell \nu \nu \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \varepsilon ) (%)</td>
<td>0.480 ( \pm ) 0.010</td>
</tr>
<tr>
<td>( f_{\text{bg}} )</td>
<td>0.102 ( \pm ) 0.002</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( \tau \to \mu \nu \nu \gamma )</th>
<th>( \tau \to \ell \nu \nu \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Photon efficiency</td>
<td>1.8</td>
</tr>
<tr>
<td>Particle identification</td>
<td>1.5</td>
</tr>
<tr>
<td>Background evaluation</td>
<td>0.9</td>
</tr>
<tr>
<td>BF</td>
<td>0.7</td>
</tr>
<tr>
<td>Luminosity and cross section</td>
<td>0.6</td>
</tr>
<tr>
<td>MC statistics</td>
<td>0.5</td>
</tr>
<tr>
<td>Selection criteria</td>
<td>0.5</td>
</tr>
<tr>
<td>Trigger selection</td>
<td>0.5</td>
</tr>
<tr>
<td>Track reconstruction</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2.8</strong></td>
</tr>
</tbody>
</table>

\( \mathcal{B}(\tau \to \mu \nu \nu \gamma)[E_{\gamma}^* > 10 \text{ MeV}] = (3.69 \pm 0.03 \pm 0.10) \times 10^{-3} \)

\( \mathcal{B}(\tau \to \ell \nu \nu \gamma)[E_{\gamma}^* > 10 \text{ MeV}] = (1.847 \pm 0.015 \pm 0.052) \times 10^{-2} \)

Measured branching ratios agree with the LO predictions (\( \mathcal{B}(\mu \nu \nu \gamma) = 3.663 \times 10^{-3} \), \( \mathcal{B}(\ell \nu \nu \gamma) = 1.834 \times 10^{-2} \)), however the LO+NLO prediction for the \( \tau \to \ell \nu \nu \gamma \) (\( \mathcal{B}(\ell \nu \nu \gamma) = 1.645 \times 10^{-2} \)) differs from the experimental result by 3.5\( \sigma \). It is important to embed NLO corrections to the MC generator (TAUOLA) of the radiative leptonic decay. Also background from the doubly-radiative leptonic decays should be properly studied and subtracted.

**M. Fael, L. Mercolli and M. Passera, JHEP 1507 (2015) 153.**
Tau decays into 5 leptons (I)

\[
\begin{align*}
\tau^- & \rightarrow e^- + e^- + e^+ + \nu_e + \bar{\nu}_e \\
\tau^- & \rightarrow \mu^- + \mu^- + \nu_\mu + \bar{\nu}_\mu
\end{align*}
\]


<table>
<thead>
<tr>
<th>Mode</th>
<th>(B_{\text{theory}})</th>
<th>(B_{\text{CLEO}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^\pm e^+ e^- 2\nu)</td>
<td>((4.21 \pm 0.01) \times 10^{-5})</td>
<td>((2.7^{+1.6}_{-1.2}) \times 10^{-5})</td>
</tr>
<tr>
<td>(\mu^\pm e^+ e^- 2\nu)</td>
<td>((1.984 \pm 0.004) \times 10^{-5})</td>
<td>(&lt; 3.2 \times 10^{-5}) (90% CL)</td>
</tr>
<tr>
<td>(e^\pm \mu^+ \mu^- 2\nu)</td>
<td>((1.247 \pm 0.001) \times 10^{-7})</td>
<td></td>
</tr>
<tr>
<td>(\mu^\pm \mu^+ \mu^- 2\nu)</td>
<td>((1.183 \pm 0.001) \times 10^{-7})</td>
<td></td>
</tr>
</tbody>
</table>


\[
\frac{d\Gamma(\tau)}{dP_S} = Q_{LL}d_1 + Q_{LR}d_2 + Q_{RL}d_3 + Q_{RR}d_4 + B_{RL}d_5 + B_{LR}d_6
\]

Up to now \(Q_{LL}, Q_{LR}, Q_{RL}, Q_{RR}, B_{RL}, B_{LR}\) were measured only in muon decays \((\mu^- \rightarrow e^- e^- e^+ \nu_\mu \bar{\nu}_e)\) with the accuracy of about 10 ÷ 20%.

Recently, analysis of 5-lepton \(\tau\) decays has been started at Belle.
Tau decays into 5 leptons (II)

\[ \tau^- \rightarrow e^- e^- e^- \bar{\nu}_e \nu_\tau \]

\[ \tau^- \rightarrow e^- \mu^- \bar{\nu}_e \nu_\tau \]

Detection efficiency, %

\[ 1.769 \pm 0.004 \]

\[ 1.204 \pm 0.003 \]

\[ 3.561 \pm 0.006 \]

\[ 1.674 \pm 0.004 \]

Main background(s)

\[ e^- \bar{\nu}_e \nu_\tau \gamma, \]

\[ \pi^- \pi^0 \nu_\tau \]

\[ \pi^- \pi^0 (\rightarrow e^+ e^- \gamma) \nu_\tau, \]

\[ \pi^- \pi^0 \pi^0 \nu_\tau \]

Expected number of signal events

\[ 1300 \]

\[ 430 \]

\[ 8 \]

\[ 4 \]

Fraction of the signal, %

\[ 47 \]

\[ 50 \]

\[ 37 \]

\[ 16 \]

The study is performed as a blinded analysis. Selection criteria were elaborated. The expected background in the signal region is estimated. Systematic uncertainties are under investigation.
Michel parameters can be measured in two ways: in the study of the dynamics and from the measurement of the branching fraction:

\[
\frac{B_{\text{exp}} - B_{\text{SM}}}{B_{\text{SM}}} = (Q_{LL} - 1) + \alpha_{LR} Q_{LR} + \alpha_{RL} Q_{RL} + \alpha_{RR} Q_{RR} + \beta_{RL} B_{RL} + \beta_{LR} B_{LR}
\]

Recently, the possibility to measure anomalous magnetic moment of \( \tau \), \( a_\tau \) was discussed in arXiv:1711.01393

\[
B = B_0 + a_\tau B_1.
\]
Lepton universality in the SM

\[ g_e = g_\mu = g_\tau \]

\[
\begin{align*}
\Gamma(L^- \rightarrow \ell^- \bar{\nu}_\ell \nu_L(\gamma)) &= \frac{\mathcal{B}(L^- \rightarrow \ell^- \bar{\nu}_\ell \nu_L(\gamma))}{\tau_L} = \frac{g_L^2 g_\ell^2}{32 M_W^4} \frac{m_L^5}{192 \pi^3} F_{\text{corr}}(m_L, m_\ell) \\
F_{\text{corr}}(m_L, m_\ell) &= f(x) \left( 1 + \frac{3}{5} \frac{m_\ell^2}{M_W^2} \right) \left( 1 + \frac{\alpha(m_L)}{2 \pi} \left( \frac{25}{4} - \pi^2 \right) \right) \\
f(x) &= 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \ x = m_\ell/m_L
\end{align*}
\]

\[
\begin{align*}
\mathcal{B}(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu(\gamma)) &= 1 \\
\frac{g_\tau}{g_e} &= \sqrt{\mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} F_{\text{corr}}(m_\mu, m_e)}, \quad \frac{g_\tau}{g_e} = 1.0029 \pm 0.0015 \quad \text{(HFAG2017)} \\
\frac{g_\tau}{g_\mu} &= \sqrt{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma)) \frac{\tau_\mu}{\tau_\tau} \frac{m_\mu^5}{m_\tau^5} F_{\text{corr}}(m_\mu, m_e)}, \quad \frac{g_\tau}{g_\mu} = 1.0010 \pm 0.0015 \quad \text{(HFAG2017)} \\
\frac{g_\mu}{g_e} &= \sqrt{\frac{\mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau(\gamma))}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau(\gamma))} \frac{F_{\text{corr}}(m_\tau, m_e)}{F_{\text{corr}}(m_\tau, m_\mu)}}, \quad \frac{g_\mu}{g_e} = 1.0019 \pm 0.0014 \quad \text{(HFAG2017)}
\end{align*}
\]
Test of lepton universality at \textit{BABAR} (I)

$$\int L dt = 467 \text{ fb}^{-1}$$

**Selections:**
- 4-track events with zero net charge;
- \(0.1 \sqrt{s} < E_{\text{miss}}^{\text{CMS}} < 0.7 \sqrt{s}, \mid \cos(\theta_{\text{miss}}^{\text{CMS}})\mid < 0.7\)
- thrust > 0.9, signal hemisphere: \(\ell/h(\ell = e, \mu; h = \pi, K)\), tag hemisphere: \(\tau \rightarrow \pi \pi \pi \nu\);
- signal hemisphere: \(E_{\text{extra} \gamma}^{\text{LAB}} < \{1.0, 0.5, 0.2, 0.2\} \text{ GeV for } \{e, \mu, \pi, K\}\), respectively

\[\tau \rightarrow e \nu \nu: N_{\text{sel}} = 884426, \varepsilon = (0.589 \pm 0.010)\%, \text{ purity is } (99.69 \pm 0.06)\%\]
Test of lepton universality at $BABAR$ (II)

\[ R_\mu = \frac{\mathcal{B}(\tau \rightarrow \mu \nu \nu)}{\mathcal{B}(\tau \rightarrow e \nu \nu)} = 0.9796 \pm 0.0016 \pm 0.0036 \]

\[ R_\pi = \frac{\mathcal{B}(\tau \rightarrow \pi \nu)}{\mathcal{B}(\tau \rightarrow e \nu \nu)} = 0.5945 \pm 0.0014 \pm 0.0061 \]

\[ R_K = \frac{\mathcal{B}(\tau \rightarrow K \nu)}{\mathcal{B}(\tau \rightarrow e \nu \nu)} = 0.03882 \pm 0.00032 \pm 0.00057 \]

\[ \left( \frac{g_\mu}{g_e} \right)_\tau = \sqrt{R_\mu \frac{F_{\text{corr}}(m_\tau, m_e)}{F_{\text{corr}}(m_\tau, m_\mu)}} = 1.0036 \pm 0.0020 \]

\[ \left( \frac{g_\tau}{g_\mu} \right)_h = \frac{\mathcal{B}(\tau \rightarrow h \nu_\tau)}{\mathcal{B}(h \rightarrow \mu \nu_\mu)} \frac{2m_\mu m_\tau^2}{(1 + \delta_h) m_\tau^3} \left( \frac{1 - m_\mu^2/m_\tau^2}{1 - m_h^2/m_\tau^2} \right)^2 \]

\[ \left( \frac{g_\tau}{g_\mu} \right)_\pi = 0.9856 \pm 0.0057, \quad \left( \frac{g_\tau}{g_\mu} \right)_K = 0.9827 \pm 0.0086 \]

\[ \left( \frac{g_\tau}{g_\mu} \right)_h = 0.9850 \pm 0.0054 \text{ (2.8\sigma away from SM)} \]

\[ \left( \frac{g_\tau}{g_\mu} \right)_{\tau+\pi+K} = 1.0000 \pm 0.0014 \text{ (HFAG2017)} \]
The world largest statistics of τ leptons collected by Belle and BABAR opens new era in the precision tests of the Standard Model and search for the effects of New Physics.

Complementary study of leptonic τ decays at BABAR and Belle. BABAR measured precisely the ratio of the leptonic branching ratios to test lepton universality. While Belle is working on the precision measurement of Michel parameters.

Nonzero average polarization of single τ at the Super Charm-Tau factory provides the possibility to measure all Michel parameters without tagging the opposite tau. Better systematic uncertainty can be reached due to the smaller impact of the ISR as well as smaller number of PS dimensions. Effect of the remaining contribution of the spin-spin correlation due to the unideal detection efficiency for the decays of the opposite tau should be studied with realistic MC simulation.

BABAR and Belle performed complementary study of the radiative leptonic τ decay $(\tau \rightarrow \ell \nu \nu \gamma \ (\ell = e, \mu))$:
- With the statistics of 431 fb$^{-1}$ branching fractions were measured with the relative accuracy better than 3% by BABAR:
  \[
  \mathcal{B}(\tau \rightarrow \mu \nu \nu \gamma)[E^*_\gamma > 10\text{ MeV}] = (3.69 \pm 0.03 \pm 0.10) \times 10^{-3} \\
  \mathcal{B}(\tau \rightarrow e \nu \nu \gamma)[E^*_\gamma > 10\text{ MeV}] = (1.847 \pm 0.015 \pm 0.052) \times 10^{-2}
  \]
- For the first time Belle measured Michel parameters, $\bar{\eta}$ and $\xi \kappa$ in $\tau \rightarrow \ell \nu \nu \gamma$ decays on the statistics of 703 fb$^{-1}$:
  \[
  \bar{\eta} = -1.3 \pm 1.5 \pm 0.8 \\
  \xi \kappa = 0.5 \pm 0.4 \pm 0.2
  \]

An importance of the NLO corrections and doubly-radiative decays was realized for the precision measurement of the branching ratios.

Good potential for the Super Charm-Tau factory to improve the results obtained at B factories and compete with Belle II.

Five-body leptonic decays are studied at Belle.

Good potential for the Super Charm-Tau factory to improve Belle results and compete with Belle II: discover $\tau \rightarrow \ell \mu \mu \nu \nu$ and $\tau \rightarrow \mu \mu \mu \nu \nu$ decays and measure Michel parameters.
Backup slides
\[ \rho = \frac{3}{4} - \frac{3}{4} \left( |g_{LR}^V|^2 + |g_{RL}^V|^2 + 2|g_{LR}^T|^2 + 2|g_{RL}^T|^2 + \Re (g_{LR}^S g_{LR}^{T*} + g_{RL}^S g_{RL}^{T*}) \right) \]

\[ \eta = \frac{1}{2} \Re \left( 6g_{RL}^V g_{LR}^{T*} + 6g_{LR}^V g_{RL}^{T*} + g_{RR}^S g_{LL}^V + g_{RL}^S g_{LR}^V + g_{LR}^S g_{RL}^V + g_{LL}^S g_{RR}^V \right) \]

\[ \xi = 4\Re (g_{LR}^S g_{LR}^{T*}) - 4\Re (g_{RL}^S g_{RL}^{T*}) + |g_{LL}^V|^2 + 3|g_{LR}^V|^2 - 3|g_{RL}^V|^2 - |g_{RR}^V|^2 + 5|g_{LR}^T|^2 - 5|g_{RL}^T|^2 + \frac{1}{4} |g_{LL}^S|^2 - \frac{1}{4} |g_{LR}^S|^2 + \frac{1}{4} |g_{RL}^S|^2 - \frac{1}{4} |g_{RR}^S|^2 \]

\[ \xi \delta = \frac{3}{16} |g_{LL}^S|^2 - \frac{3}{16} |g_{LR}^S|^2 + \frac{3}{16} |g_{RL}^S|^2 - \frac{3}{16} |g_{RR}^S|^2 - \frac{3}{4} |g_{LR}^T|^2 + \frac{3}{4} |g_{RL}^T|^2 + \frac{3}{4} |g_{LL}^V|^2 - \frac{3}{4} |g_{LR}^V|^2 + \frac{3}{4} \Re (g_{LR}^S g_{LR}^{T*}) - \frac{3}{4} \Re (g_{RL}^S g_{RL}^{T*}) \]

\[ \bar{\eta} = |g_{RL}^V|^2 + |g_{LR}^V|^2 + \frac{1}{8} \left( |g_{RL}^S + 2g_{RL}^T|^2 + |g_{LR}^S + 2g_{LR}^T|^2 \right) + 2 \left( |g_{RL}^T|^2 + |g_{LR}^T|^2 \right) \]

\[ \xi \kappa = |g_{RL}^V|^2 - |g_{LR}^V|^2 + \frac{1}{8} \left( |g_{RL}^S + 2g_{RL}^T|^2 - |g_{LR}^S + 2g_{LR}^T|^2 \right) + 2 \left( |g_{RL}^T|^2 - |g_{LR}^T|^2 \right) \]