Radiative Corrections for Super C-Tau Factory: Status and Perspectives

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Outline

MOTIVATION

Motivation is clear:
an advanced treatment of radiative corrections (RC) is a must for new high-precision experiments

QUESTIONS:

- What we have?
- What we need?
- What to do?

RC: general remarks

Types of effects: QED (pert.), Strong (non-pert.), Weak

Interplay of QED and Strong effects: vacuum polarization and form factors

Weak effects are small ($\sim s/M_Z^2$), but visible and sensitive to polarization.

MAGNITUDE OF QED RC

We have several small and large parameters to be used in expansions:

- $\alpha/(2\pi) \approx 0.001$
- $(\alpha/(2\pi))^2 \approx 10^{-6}$
- $L \equiv \ln(E_e^2/m_e^2) \approx 16$ the large log for $E_e^2 = 1\ \mathrm{GeV}^2$
- $(m_e^2/E_\nu^2) \ll 1$, but for μ and τ ...
- there can be other enhancement and suppression factors due to concrete experimental conditions

Knowing the experimental precision tag is crucial. E.g. for 1% precision tag we need to control all effects of the order of a few permille.

CALCULATION OF QED RC

Standard:
perturbative + re-summed in some cases + leading logs in higher orders

For most processes we have now $1 + \frac{1}{2}$ orders in α_{QED} in RC to relevant processes

Difficulties:
Keeping exact dependence on m_τ^2/s in higher orders,
Γ_τ in loops, hadronic formfactors, …

PROCESSES

- Bhabha scattering
- $e^+e^- \to 2\gamma$
- $e^+e^- \to \mu^+\mu^-, \; e^+e^- \to \tau^+\tau^-$
- $e^+e^- \to \pi^+\pi^-, \; e^+e^- \to K^+K^-, \ldots$
- radiative return

Monte Carlo generators:

`MCGPJ, BHWIDE, ALIBABA, KKMC, PHOKHARA, ...`

POLARIZED BHABHA SCATTERING (I)

Complete one-loop EW RC to (longitudinally) polarized Bhabha scattering are computed for the first time by the SANC group (D.Bardin, A.Arbuzov, S.Bondarenko, Ya.Dydyshko, L.Kalinovskaya, R.Sadykov, L.Rumyantsev)

Tuned comparisons with know results without polarization are performed, agreement is demonstrated.

Tuned comparisons with know results with polarization for tree-level contributions (Born and hard Bremsstrahlung) are also preformed.

POLARIZED BHABHA SCATTERING (II)

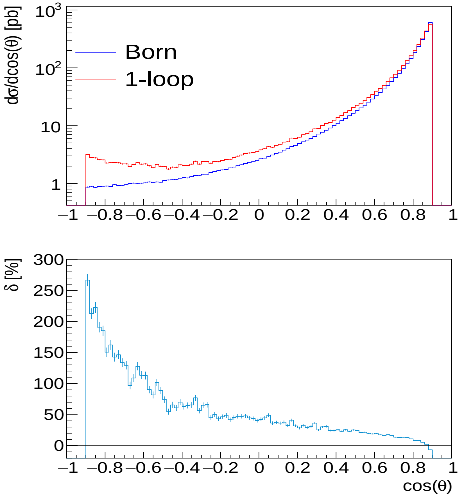
PRELIMINARY

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma^{\text{Born}}_{e^+e^-}$, pb [WHIZARD]	56.677(1)	57.774(1)	56.272(1)	59.276(1)
$\sigma^{\text{Born}}_{e^+e^-}$, pb [SANC]	56.677(1)	57.775(1)	56.272(1)	59.275(1)
$\sigma^{\text{hard}}_{e^+e^-}$, pb [WHIZARD]	48.618(4)	49.575(4)	48.742(3)	50.398(4)
$\sigma^{\text{hard}}_{e^+e^-}$, pb [SANC]	48.638(5)	49.591(5)	48.767(4)	50.438(5)

Table : Tuned comparison of SANC and WHIZARD for Born and hard bremsstrahlung cross-section of Bhabha scattering for $\sqrt{s} = 250$ GeV

POLARIZED BHABHA SCATTERING (III)

PRELIMINARY The differential cross section of Bhabha scattering as a functionof the electron scattering angle ($\sqrt{s} = 250$ GeV)



HIGHER ORDER INCLUSIVE FSR IN $e^+e^- \to l^+l^-$ (I)

[**A.Kataev**, A.Garkusha, V.Molokoedov; Preprint INR-TH-2017-007]

1. The $O(\alpha^4)$ analytical QED expression for the quantity $\sigma_{tot}(e^+e^- \to \gamma \to l_i^+ l_i^-)$ will be presented through the running QED coupling constants in the $\overline{\rm MS}$ and on-shell (OS) scheme in the massless approximation, where l_1^+=e^+, $l_2^+ = \mu^+$ and $l_3^+ = \tau^+$

2. The $\overline{\rm MS}$-scheme results will be obtained from by transforming $SU(N_c)$ QCD analytical results of calculations of

$$R^{e^+e^-}(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma \to \ hadrons)}{\sigma_{Born}(e^+e^- \to \gamma \to \mu^+\mu^-)}$$

evaluated at the α_s^2 level by Chetyrkin, Kataev, Tkachov (79) analytically and by Dine, Sapirtsein (79) numererically, at the α_s^3 level by Gorishny, Kataev, Larin (91) analytically (confirmed analytically by Surguladze, Samuel (91) , and at the α_s^4 level analytically by Baikov, Chetyrkin, Kuhn (10) (NS-contributions)+ Baikov,Chetyrkin, Kuhn, Rirbergen (12) (singlet- SI-contributions) and confirmed recently analytically by F.Herzog, B.Ruijl, T.Ueda, J.A.M.Vermaseren and A.Vogt JHEP **1708** (2017) 113

HIGHER ORDER INCLUSIVE FSR IN $e^+e^- \to l^+l^-$ (II)

The studied QED physical quantity is defined as

$$\sigma(e^+e^- \to \gamma \to l_i + l_i^-) = \frac{4\pi\alpha_{EM}^2}{3s} F(v_{l_i}) \tilde{\sigma}_{l_i}(e^+e^- \to \gamma \to l_i^+ l_i^-)$$

$$\tilde{\sigma}_{l_i}(e^+e^- \to \gamma \to l^+l^-) = 1 + \sum_{i=1}^{4} \left[r_i + \mathcal{O}\left(\frac{m_{l_i}^2}{s}\right)\right] a^i(s)$$

where $F(v_{l_i}) = v_{l_i}(3 - v_{l_i}^2)/2$ is the threshold factor, $v_{l_i} = (1 - 4m_{l_i}^2/s)^{1/2}$, $\alpha_{EM} \approx 1/137$, m_{l_i} are the pole masses of e, μ or τ-leptons respectively and

$$a(s) = \alpha(s)/\pi$$

with the QED running coupling constant $a(s) = \alpha(s)/\pi$ are defined in the $\overline{\text{MS}}$ or in the OS schemes.

HIGHER ORDER INCLUSIVE FSR IN $e^+e^- \to l^+l^-$ (III)

Analytical expressions are derived. The corresponding numerical results for different channels are

$$\tilde{\sigma}(e^+e^- \to \gamma \to e^+e^-)^{\overline{\rm MS}} = 1 + 0.75 \cdot a_{\overline{\rm MS}}(s) - 0.2667 \cdot a^2_{\overline{\rm MS}}(s) - 1.124 \cdot a^3_{\overline{\rm MS}}(s) + 4.5253 \cdot a^4_{\overline{\rm MS}}(s)$$
$$\tilde{\sigma}(e^+e^- \to \gamma \to e^+e^-)^{OS} = 1 + 0.75 \cdot a_{OS}(s) - 0.2667 \cdot a^2_{OS}(s) - 0.4207 \cdot a^3_{OS}(s) + 3.2663 \cdot a^4_{OS}(s)$$

$$\tilde{\sigma}(e^+e^- \to \gamma \to \mu^+\mu^-)^{\overline{\rm MS}} = 1 + 0.75 \cdot a_{\overline{\rm MS}}(s) - 0.3496 \cdot a^2_{\overline{\rm MS}}(s) - 2.826 \cdot a^3_{\overline{\rm MS}}(s) + 8.9899 \cdot a^4_{\overline{\rm MS}}(s)$$
$$\tilde{\sigma}(e^+e^- \to \gamma \to \mu^+\mu^-)^{OS} = 1 + 0.75 \cdot a_{OS}(s) - 0.3496 \cdot a^2_{OS}(s) - 1.14301 \cdot a^3_{OS}(s) + 4.39172 \cdot a^4_{OS}(s)$$

$$\tilde{\sigma}(e^+e^- \to \gamma \to \tau^+\tau^-)^{\overline{\rm MS}} = 1 + 0.75 \cdot a_{\overline{\rm MS}}(s) - 0.6126 \cdot a^2_{\overline{\rm MS}}(s) - 5.6765 \cdot a^3_{\overline{\rm MS}}(s) + 16.619 \cdot a^4_{\overline{\rm MS}}(s)$$
$$\tilde{\sigma}(e^+e^- \to \gamma \to \tau^+\tau^-)^{OS} = 1 + 0.75 \cdot a_{OS}(s) - 0.6126 \cdot a^2_{OS}(s) - 3.6567 \cdot a^3_{OS}(s) + 6.6315 \cdot a^4_{OS}(s)$$

Conclusion: in the OS scheme starting from $O(a^3)$-level PT series have smaller coefficients, then in the $\overline{\rm MS}$ scheme. That is useful for estimating theoretical error bars.

$e^+e^- \to$ HADRONS AT HIGHER LOOPS

One more example of theoretical achievements:

[A.V. Nesterenko, Electron-positron annihilation into hadrons at the higher-loop levels, EPJC 77 (2017) no.12, 844.]

The strong corrections to the R **ratio** of electron-positron annihilation into hadrons are studied at the higher-loop levels. Specifically, the essentials of continuation of the spacelike perturbative results into the timelike domain are elucidated. The derivation of a general form of the commonly employed approximate expression for the R ratio (which constitutes its truncated re-expansion at high energies) is delineated, the appearance of the pertinent π^2 terms is expounded, and their basic features are examined. It is demonstrated that the validity range of such approximation is strictly limited to $\sqrt{s}/\Lambda_{QCD} > \exp(\pi/2) \approx 4.81$ and that it converges rather slowly when the energy scale approaches this value. The spectral function required for the proper calculation of the R-ratio is explicitly derived and its properties at the higher-loop levels are studied. The developed method of calculation of the spectral function enables one to obtain the explicit expression for the latter at an arbitrary loop level. By making use of the derived spectral function the proper expression for the R ratio is calculated up to the five-loop level and its properties are examined. It is shown that the loop convergence of the proper expression for the R ratio is better than that of its commonly employed approximation. The impact of the omitted higher-order π^2 terms on the latter is also discussed.

WHAT WE HAVE?

- We already have a lot
- There are several Monte Carlo codes
- There are new results on higher-order contributions
- There are new advanced methods of theoretical calculations
- There are new computer techniques: adaptive Monte Carlo and powerful computers

WHAT WE NEED?

Presumably,

- We need to take into account polarization
- We need an advanced treatment of higher-order corrections, e.g. complete two-loop result in some cases, and taking into account masses
- We need to re-evalute form factors in hadron-photon interactions
- We need a new Monte Carlo event genertor which will include everything :)

WHAT TO DO?

- Set clear requirements on the precision tags for particular classes of processes (e.g. Bhabha, $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to \tau^+\tau^-$, $e^+e^- \to D\bar{D}$ etc.)
- Attract the attention of the "RC community", in particular of the Radio MonteCarlo WG
- Creation of a special **working group** on RC for the c-tau factory might be worth doing

Thank You
for attention!