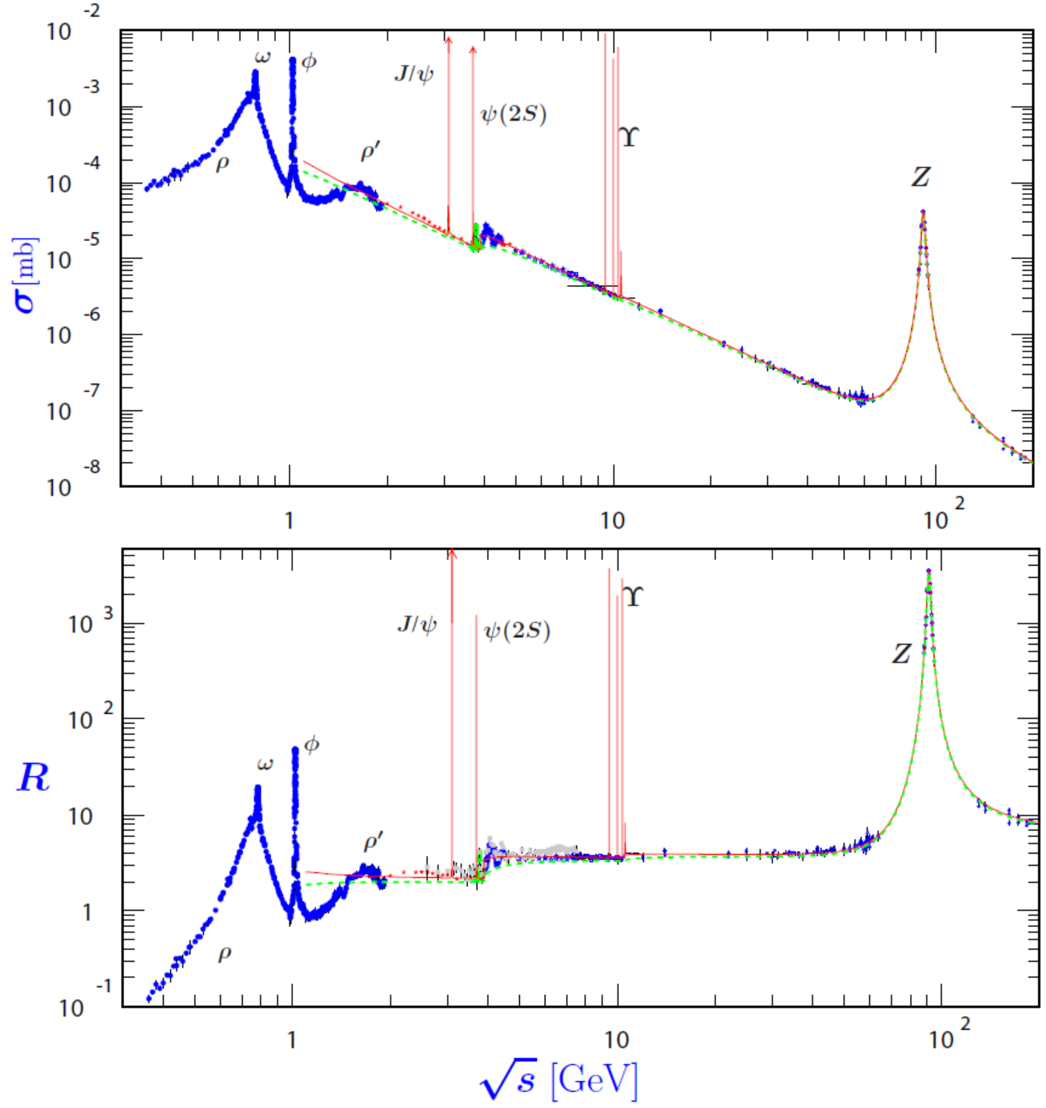
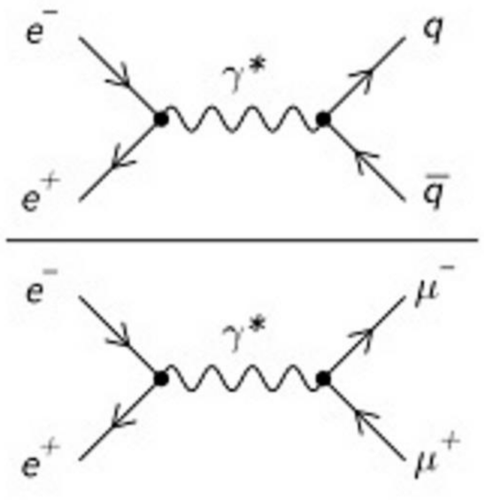
Статус измерения R на низкой энергии

23.04.2025

Motivation of R measurement



$$R = \frac{\sigma(e^-e^+ \rightarrow hadrons)}{\sigma(e^-e^+ \rightarrow \mu^-\mu^+)} \approx$$

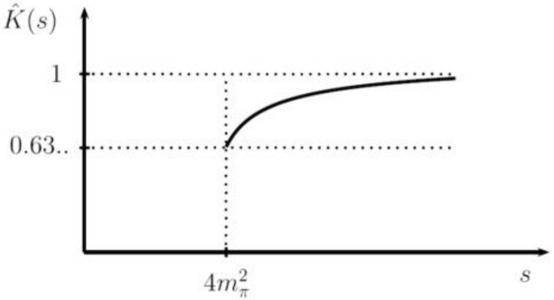


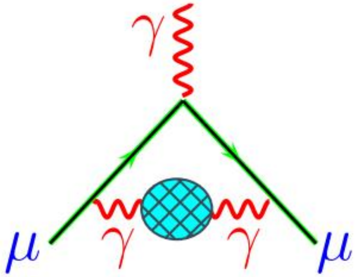
R(s) is used to determine:

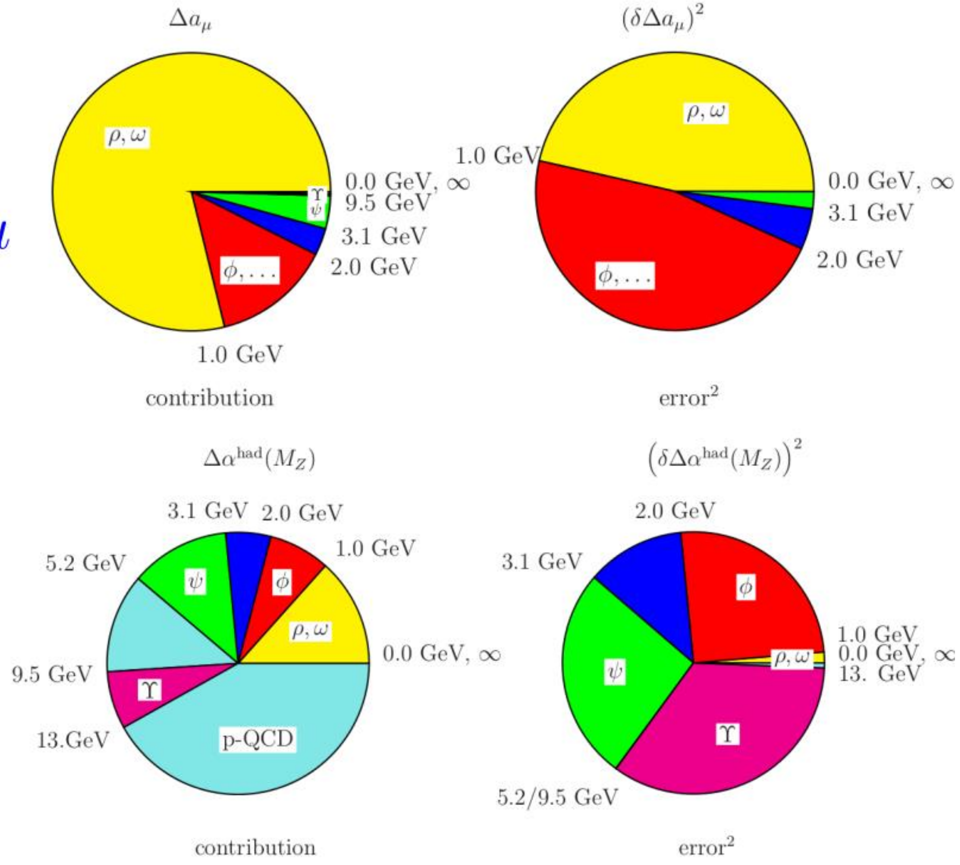
- $\alpha_s(s)$
- $(g_\mu - 2)/2$
- $\alpha\ (M_Z^2)$
- m_q

Вклад R в измерение a_μ и $\alpha(M_Z^2)$

$$a_\mu^{exp} = (g_\mu - 2)/2$$







$$a_\mu^{LO\ VP} = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{\hat{K}(s)R(s)}{s^2}ds$$

$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}$$

$$\Delta\alpha = \sum_f$$

$$= \Delta\alpha_{\mathbf{lep}}(s) + \Delta\alpha_{\mathbf{had}}(s)$$

A. Blondel и др. arXiv:1905.05078.

$$\Delta\alpha^{(5)}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi}\,\mathrm{Re}\int_{4m_\pi^2}^{\infty}\frac{R(s)ds}{s(s-M_Z^2-i\epsilon)}$$

Predictions

Naive quark model:

$$R_0(s) = \frac{\sigma(e^-e^+ \to hadrons)}{\sigma(e^-e^+ \to \mu^-\mu^+)} = N_c \sum e_q^2$$

At energy $3.77 \le \sqrt{s} \le 10.58$ GeV (u, d, s, c) : $R_0 \approx {}^{10}/_{3}$

pQCD in 3-loops: $R(s) = R_0(s)\left(1 + C_1 \frac{\alpha_s}{\pi} + C_2\left(\frac{\alpha_s}{\pi}\right)^2 + C_3\left(\frac{\alpha_s}{\pi}\right)^3 + C_4\left(\frac{\alpha_s}{\pi}\right)^4\right)$

At $n_f = 4$: $C_1 = 1, C_2 = 1.525, C_3 = -11.686$, C_4 = -89.822

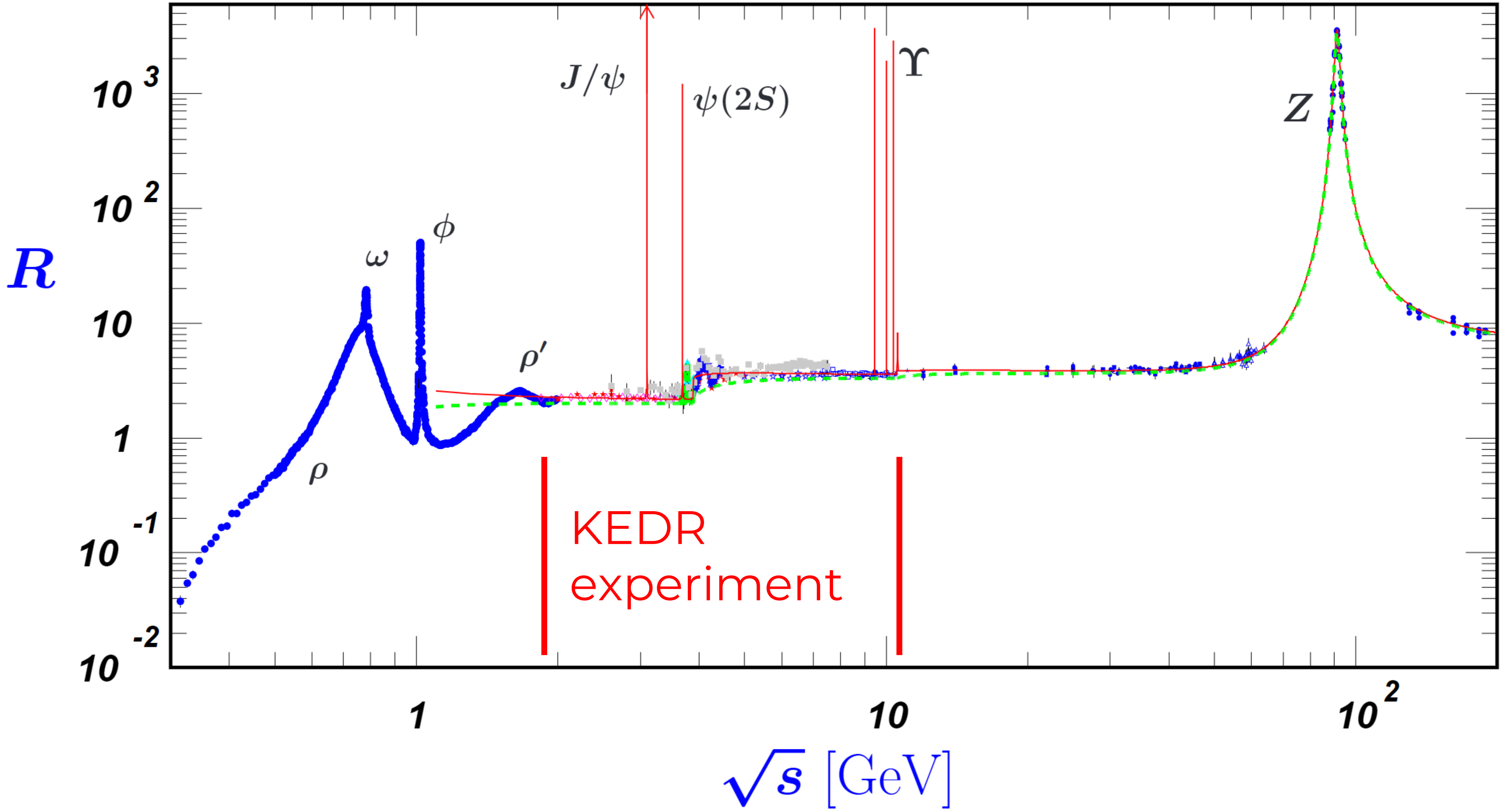
P. A. Baikov et al. Nucl. And Part. *Phys. Proceed.* 261-262 (2015)

$$\alpha_s = \frac{1}{\beta_0 L} - \frac{b_1}{(\beta_0 L)^2} \ln L + \frac{1}{(\beta_0 L)^3}\left[b_1^2\left(\ln^2 L - \ln L - 1\right) + b_2\right] + \\ + \frac{1}{(\beta_0 L)^4}\left[b_1^3\left(-\ln^3 L + \frac{5}{2}\ln^2 L + 2\ln L - \frac{1}{2}\right) - 3b_1 b_2 \ln L + \frac{b_3}{2}\right]$$

At $n_f = 4$: $\beta_0 = 2.083, b_1 = -1.540$, $b_2 = 3.048$, $b_3 = 179.558$; $L = \ln\frac{s}{\Lambda^2}$

Chetyrkin, Kniehl, Steinhauser, Nucl. Phys. B 510 (1998) 61

R measurements



R measurement between 1.8 and 3.8 GeV at KEDR -1

$\sqrt{s}$, GeV	N_{points}	$\int Ldt, pb^{-1}$	Unc., %	Ref.
1.84 – 3.05	13	0.66	≤ 3.9 total (≈2.4 syst.)	V.V. Anashin. Phys.Lett. B 770 (2017) 174
3.08 – 3.72	9	2.7	≤ 2.6 total (≈1.9 syst.)	V.V. Anashin. Phys.Lett. B 788 (2019) 42

Systematic uncertainties (KEDR)

Source	Syst. uncertainty, %		
	Scan 1 and 2 (2010-2011)	Scan 2014-2015	Correlated
Luminosity	1.1	0.9	0.4
Rad. corr.	$0.4 \div 0.6$	$0.5 \div 0.8$	$0.2 \div 0.4$
uds simulation	$1.3 \div 2.0$	1.1	0.9
Track reconstruction	0.5	0.4	–
J/ψ	$0.1 \div 2.7$	$0.1 \div 1.8$	–
$\psi(2S)$ (at 3.72 GeV)	1.4	1.1	–
l^+l^-	$0.1 \div 0.2$	$0.3 \div 0.4$	$0.1 \div 0.2$
e^+e^-X	$0.1 \div 0.2$	0.1	0.1
Trigger	0.2	0.2	0.2
Nuclear interaction	0.2	0.2	0.2
Machine background	$0.5 \div 1.1$	$0.4 \div 0.8$	–
Cuts	0.6	0.6	–
Total	$2.1 \div 3.6$ (correlated $1.8 \div 2.5$)	$1.9 \div 2.7$	1.1

Systematic uncertainties (BES3)

TABLE I. Summary of systematic uncertainties (in percent) at each c.m. energy, where the total uncertainty is the sum of the individual ones in quadrature. Uncertainties from the last four sources are correlated between the energy points.

$\sqrt{s}$ (GeV)	Event selection	QED background	Beam background	Luminosity	Trigger efficiency	Signal model	ISR correction	Total
2.2324	0.41	0.23	0.28	0.80	0.10	0.60	1.15	1.62
2.4000	0.55	0.27	0.15	0.80	0.10	1.11	1.10	1.87
2.8000	0.58	0.28	0.34	0.80	0.10	1.97	1.06	2.48
3.0500	0.61	0.33	0.41	0.80	0.10	1.76	1.01	2.33
3.0600	0.60	0.34	0.48	0.80	0.10	1.84	1.00	2.39
3.0800	0.61	0.35	0.35	0.80	0.10	1.31	1.05	2.02
3.4000	0.65	0.33	0.16	0.80	0.10	1.86	1.24	2.49
3.5000	0.60	0.35	0.62	0.80	0.10	2.05	1.16	2.66
3.5424	0.61	0.37	0.01	0.80	0.10	2.05	1.14	2.58
3.5538	0.66	0.31	0.39	0.80	0.10	2.22	1.13	2.74
3.5611	0.74	0.34	0.34	0.80	0.10	2.28	1.12	2.81
3.6002	0.66	0.33	0.38	0.80	0.10	2.27	1.09	2.77
3.6500	0.53	0.35	0.69	0.80	0.10	2.28	1.13	2.83
3.6710	0.61	0.42	0.63	0.80	0.10	2.23	1.04	2.77

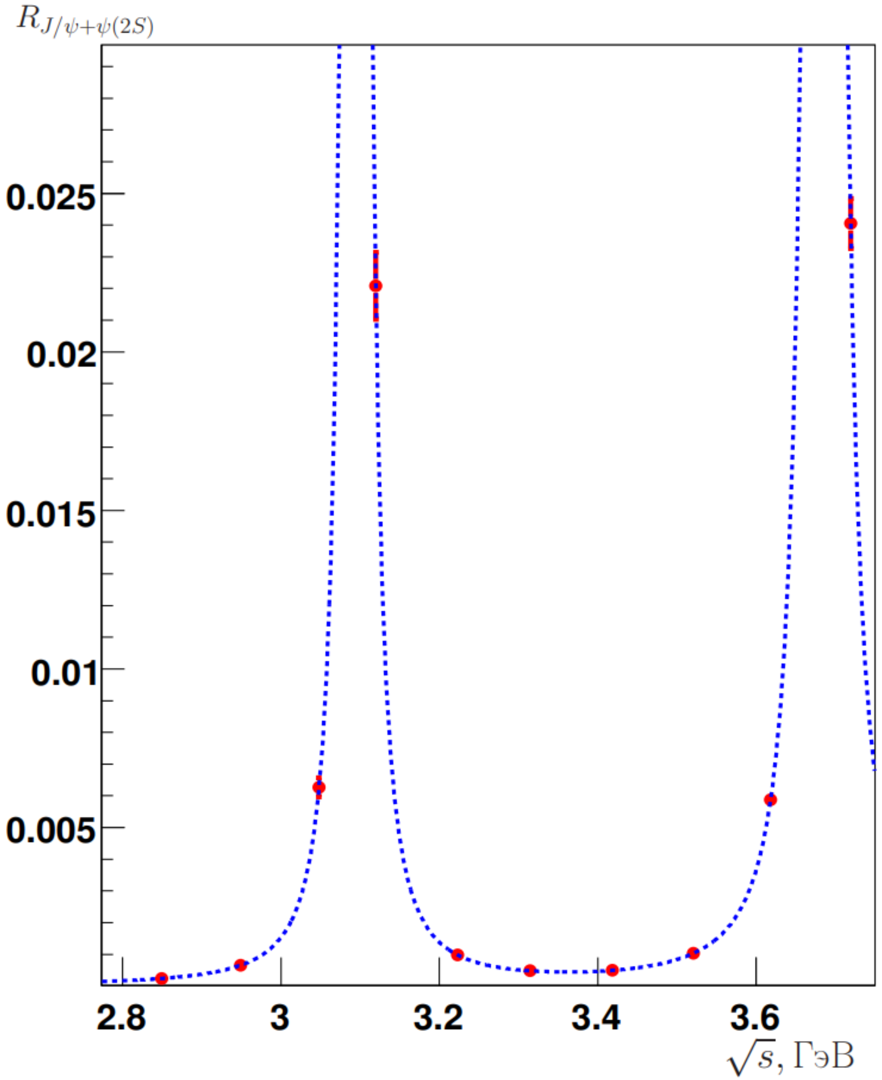
BES3 results

TABLE II. Summary of primary quantities mentioned in Eq. (1) and the measured R value for each c.m. energy, where the uncertainties are statistical and systematic respectively.

$\sqrt{s}$ (GeV)	$N_{\rm had}^{\rm obs}$	$N_{\rm bkg}$	$\sigma^0_{\mu\mu}$ (nb)	$\mathcal{L}_{\rm int}$ (pb^{-1})	$\varepsilon_{\rm had}$ (%)	$1+\delta$	R
2.2324	83227	2041	17.427	2.645	64.45	1.195	$2.286 \pm 0.008 \pm 0.037$
2.4000	96627	2331	15.079	3.415	67.29	1.204	$2.260 \pm 0.008 \pm 0.042$
2.8000	83802	2075	11.078	3.753	72.25	1.219	$2.233 \pm 0.008 \pm 0.055$
3.0500	283822	7719	9.337	14.89	73.91	1.193	$2.252 \pm 0.004 \pm 0.052$
3.0600	282467	7683	9.276	15.04	73.88	1.183	$2.255 \pm 0.004 \pm 0.054$
3.0800	552435	15433	9.156	31.02	73.98	1.123	$2.277 \pm 0.003 \pm 0.046$
3.4000	32202	843	7.513	1.733	74.81	1.382	$2.330 \pm 0.014 \pm 0.058$
3.5000	62670	1691	7.090	3.633	75.32	1.351	$2.327 \pm 0.010 \pm 0.062$
3.5424	145303	3872	6.921	8.693	75.58	1.341	$2.319 \pm 0.006 \pm 0.060$
3.5538	92996	2469	6.877	5.562	75.50	1.338	$2.342 \pm 0.008 \pm 0.064$
3.5611	64650	2477	6.849	3.847	75.50	1.337	$2.338 \pm 0.010 \pm 0.066$
3.6002	159644	9817	6.701	9.502	75.73	1.328	$2.339 \pm 0.006 \pm 0.065$
3.6500	78730	6168	6.519	4.760	76.00	1.308	$2.352 \pm 0.009 \pm 0.067$
3.6710	75253	6461	6.445	4.628	76.11	1.260	$2.405 \pm 0.010 \pm 0.067$

Результаты измерения величины R

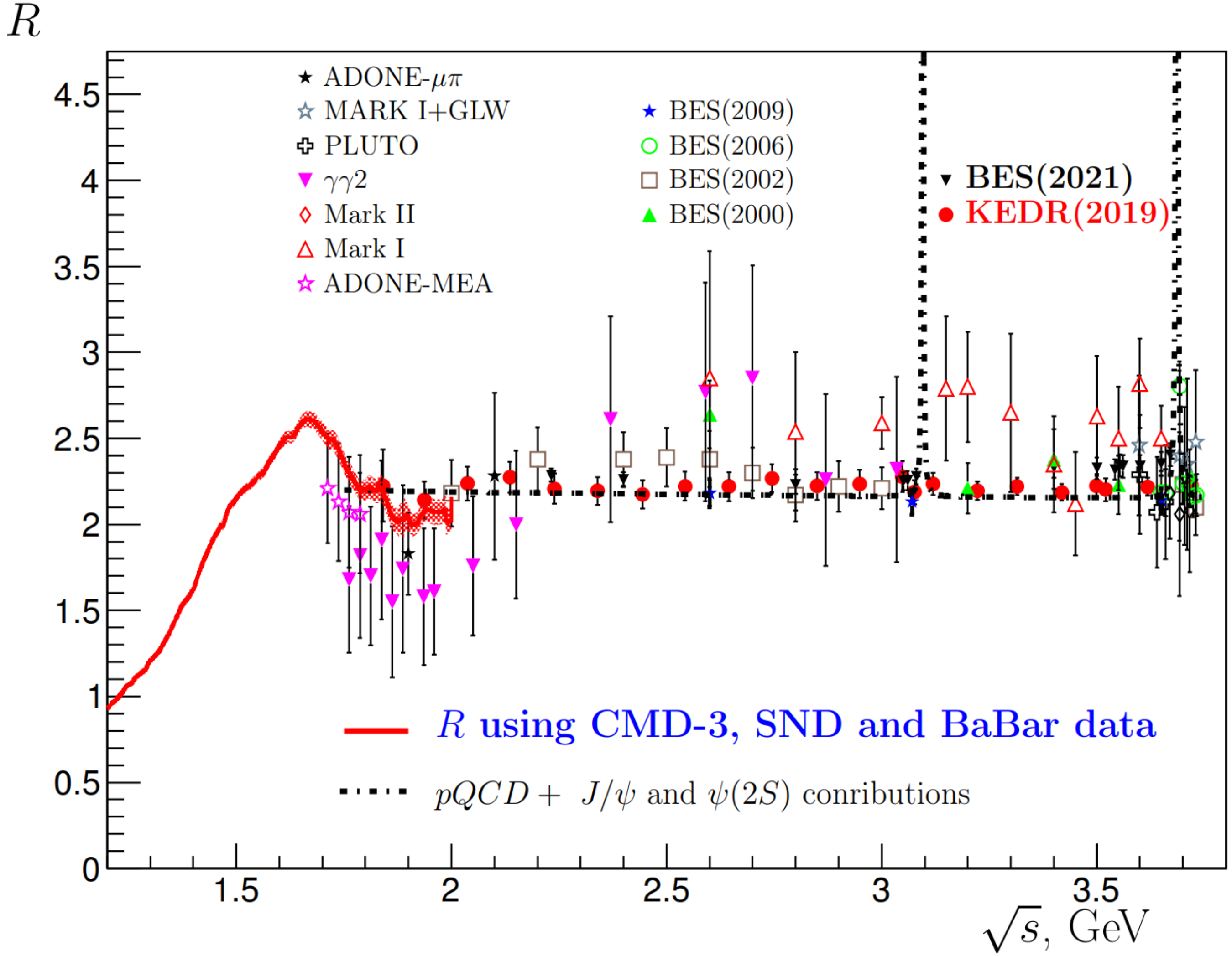
$\sqrt{s}$, ГэВ	$R_{uds}(s)\{R(s)\}$	$\frac{\delta R}{R}\left(\frac{\delta R_{syst}}{R}\right)$,%
1.841	$2.226 \pm 0.139 \pm 0.158$	9.5(7.1)
1.937	$2.141 \pm 0.081 \pm 0.073$	5.1(3.4)
2.037	$2.238 \pm 0.068 \pm 0.072$	4.4(3.2)
2.134	$2.275 \pm 0.072 \pm 0.055$	4.0(2.4)
2.239	$2.208 \pm 0.069 \pm 0.053$	3.9(2.4)
2.340	$2.194 \pm 0.064 \pm 0.048$	3.7(2.2)
2.444	$2.175 \pm 0.067 \pm 0.048$	3.8(2.2)
2.543	$2.222 \pm 0.070 \pm 0.047$	3.8(2.1)
2.645	$2.220 \pm 0.069 \pm 0.049$	3.8(2.2)
2.745	$2.269 \pm 0.065 \pm 0.050$	3.6(2.2)
2.850	$2.223 \pm 0.065 \pm 0.047$	3.6(2.1)
2.949	$2.234 \pm 0.064 \pm 0.051$	3.7(2.3)
3.048	$2.278 \pm 0.075 \pm 0.048$	3.9(2.3)
3.077	$2.188 \pm 0.056 \pm 0.042$	3.2(2.1)
3.120	$2.212\{2.235\} \pm 0.042 \pm 0.049$	2.9(2.2)
3.223	$2.194\{2.195\} \pm 0.040 \pm 0.035$	2.4(1.6)
3.315	$2.219\{2.219\} \pm 0.035 \pm 0.035$	2.2(1.6)
3.418	$2.185\{2.185\} \pm 0.032 \pm 0.035$	2.2(1.6)
3.500	$2.224\{2.224\} \pm 0.054 \pm 0.040$	3.0(1.8)
3.521	$2.200\{2.201\} \pm 0.050 \pm 0.044$	3.0(2.0)
3.618	$2.212\{2.218\} \pm 0.038 \pm 0.035$	2.3(1.6)
3.720	$2.204\{2.228\} \pm 0.039 \pm 0.042$	2.6(1.9)



Используя параметры J/ψ- и $\psi(2S)$-резонансов, находим

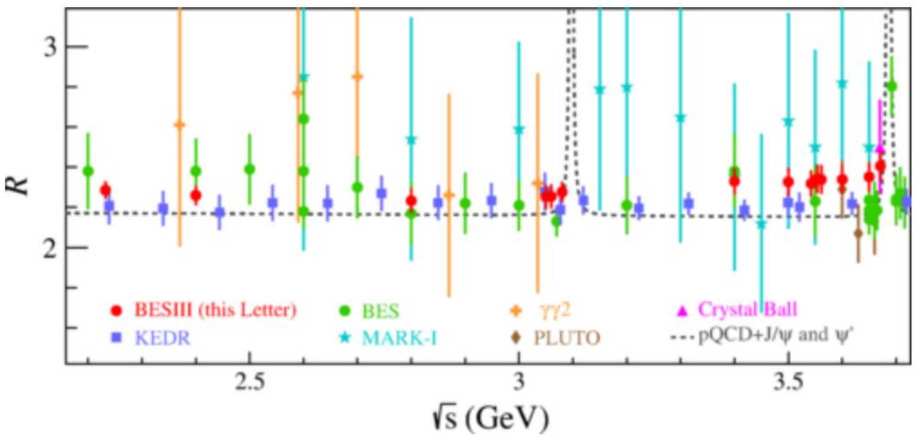
$R_{uds}(s) + R_{J/\psi+\psi(2S)} \Longrightarrow R(s)$

R measurement between 1.8 and 3.8 GeV at KEDR - 2



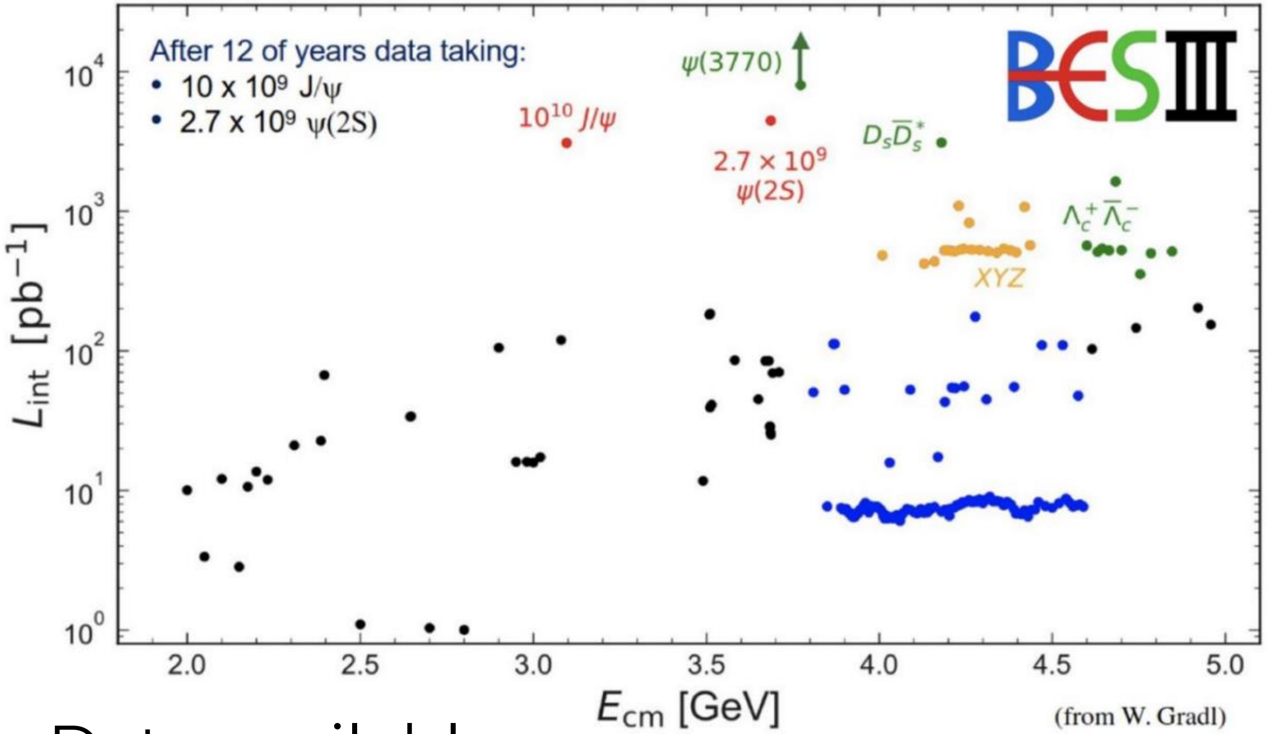
V.V. Anashin. Phys.Lett. B 770 (2017) 174

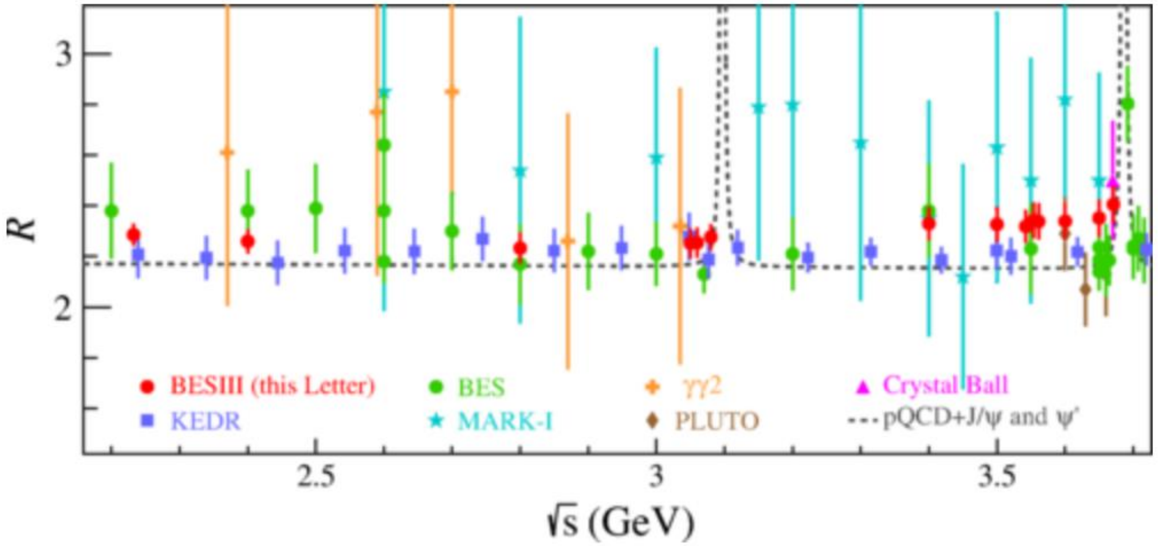
V.V. Anashin. Phys.Lett. B 788 (2019) 42



M. Ablikim *et al.* (BESIII)
Phys. Rev. Lett. **128**, 062004

R measurement at BESIII





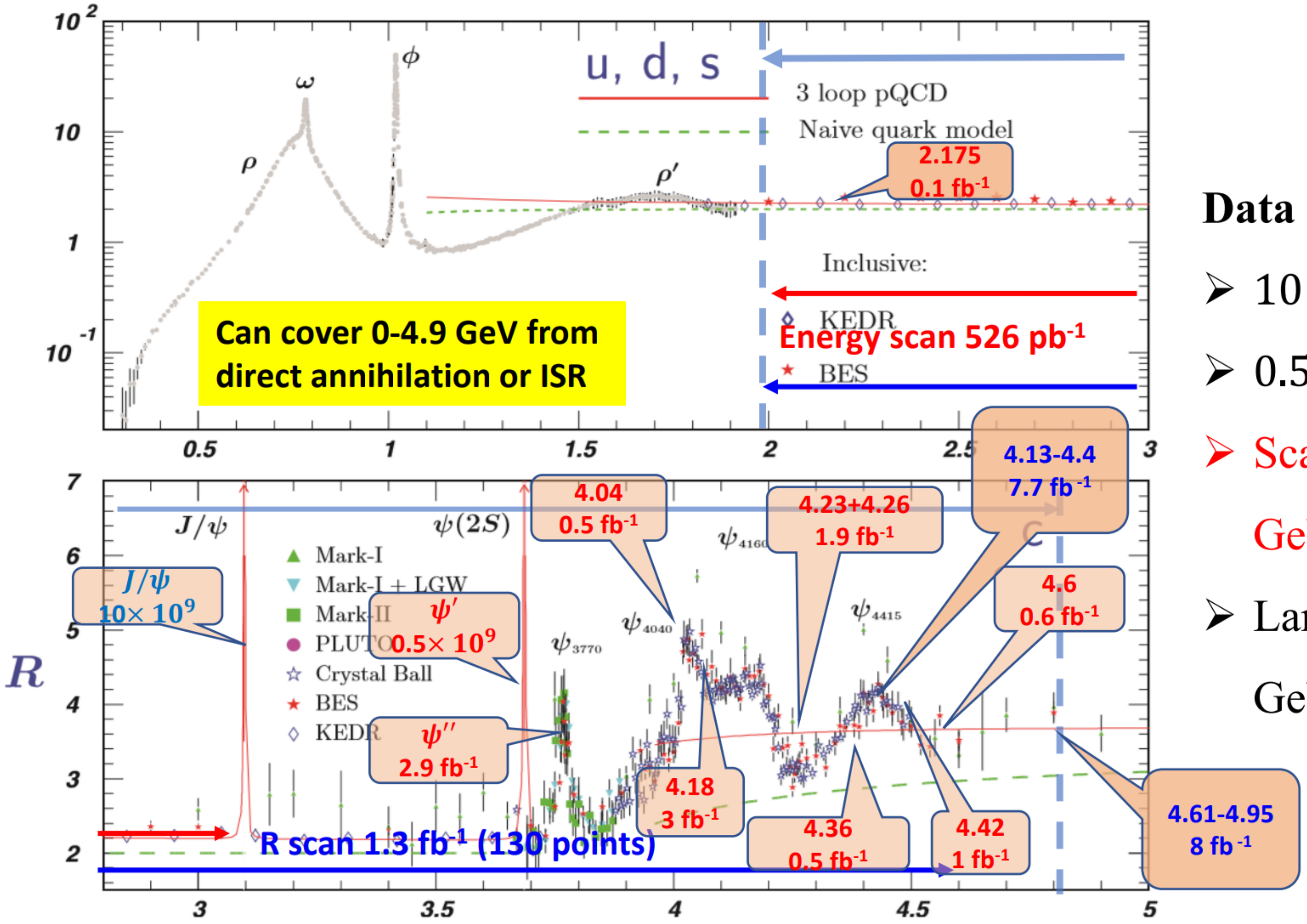
Data available:
21 energy points
2.00 – 3.08 GeV
~ 550 pb^{-1}

3.85 – 4.59 GeV
104 energy points
~ 800 pb^{-1}

Data analyzed:
2.23 – 3.67 GeV
10 energy points
~110 pb^{-1}

M. Ablikim *et al.* (BESIII)
Phys. Rev. Lett. **128**, 062004

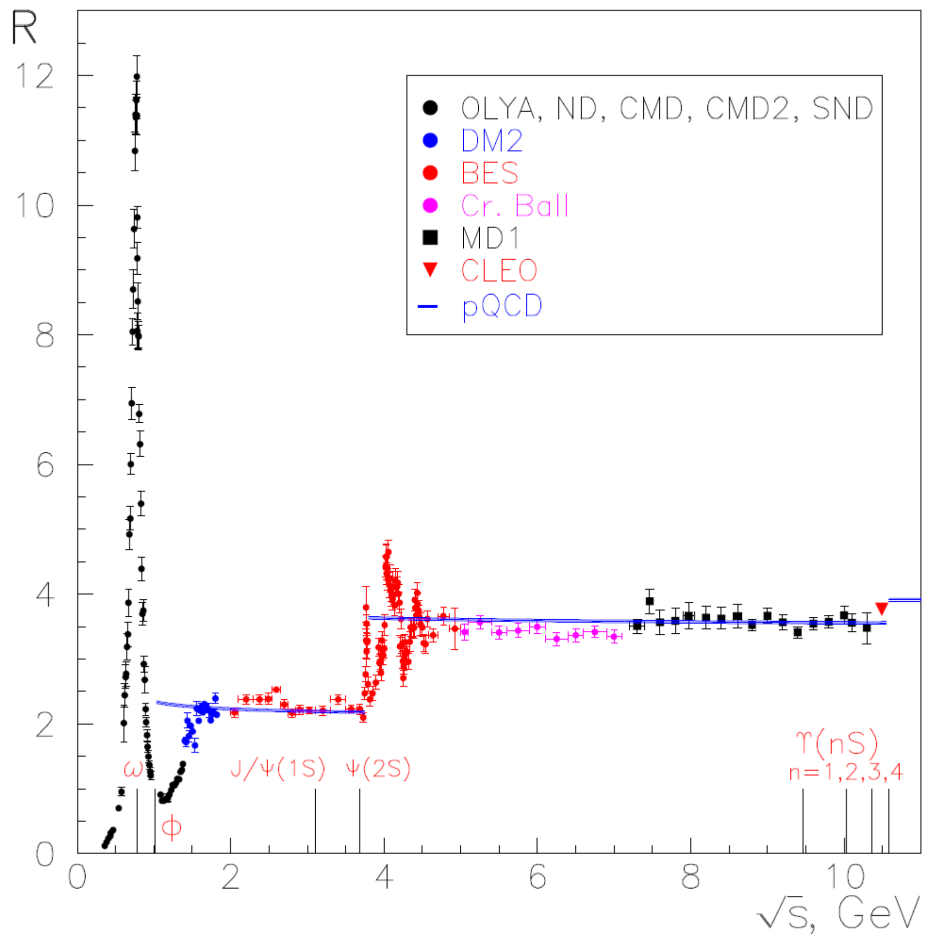
Data samples collected at BESIII

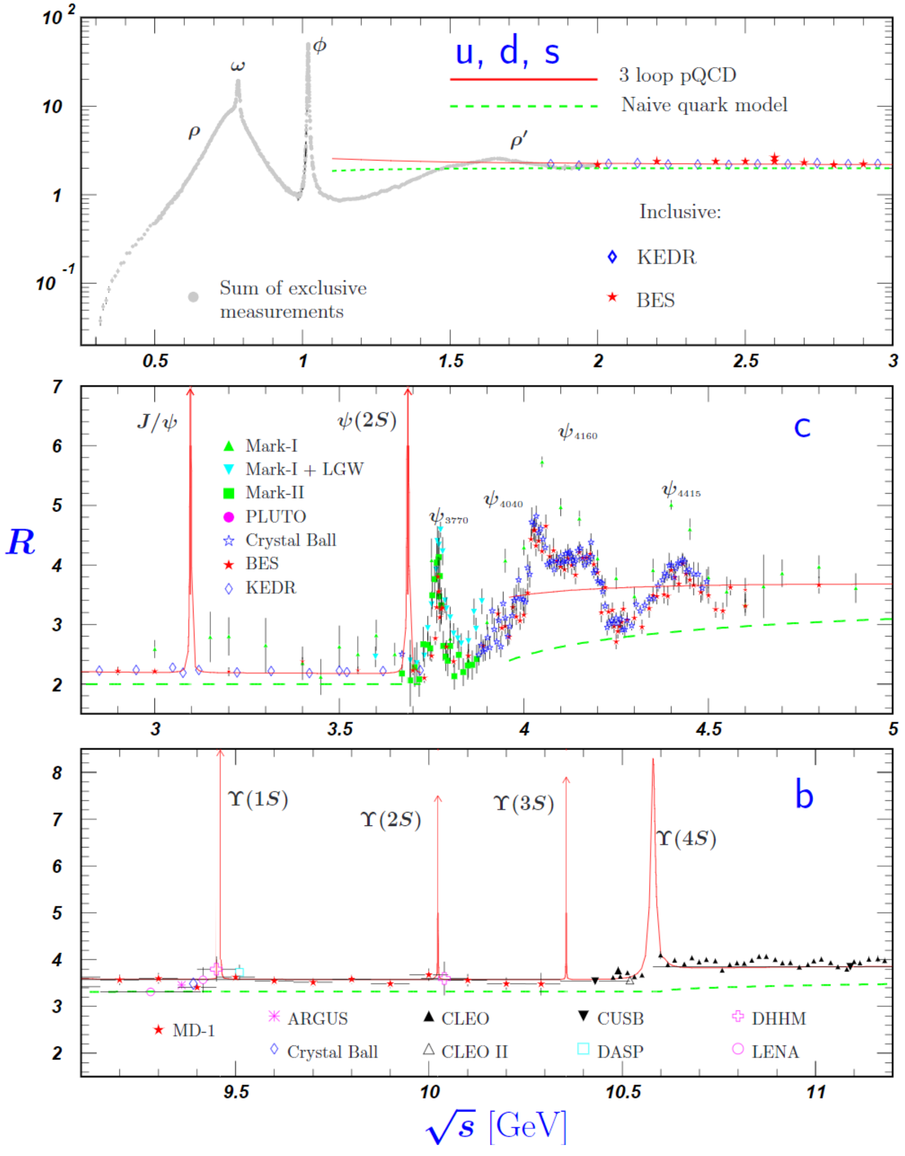


Data sets collected so far include:

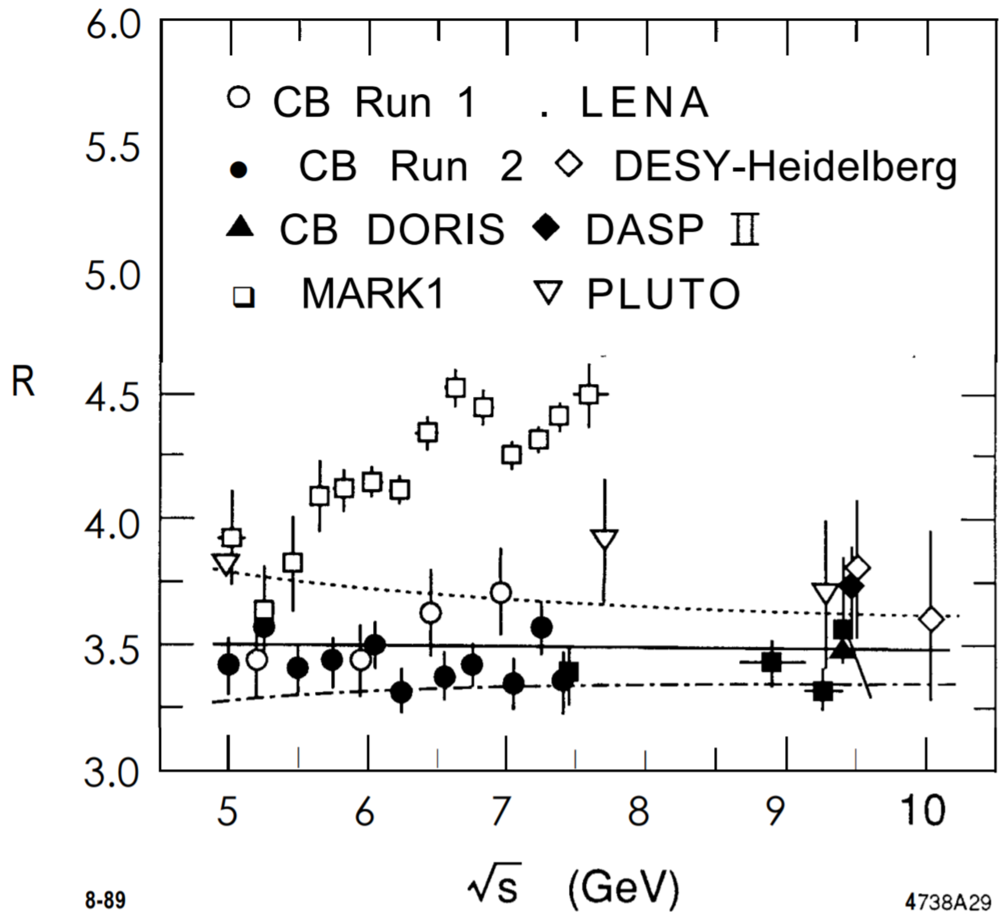
- 10×10^9 J/ψ events
- 0.5×10^9 ψ' events
- Scan data [2.0, 3.08] GeV; [3.735, 4.600] GeV, 130 energy points, about 2.0 fb^{-1}
- Large data sets for XYZ study above 4.0 GeV about 22 fb^{-1}

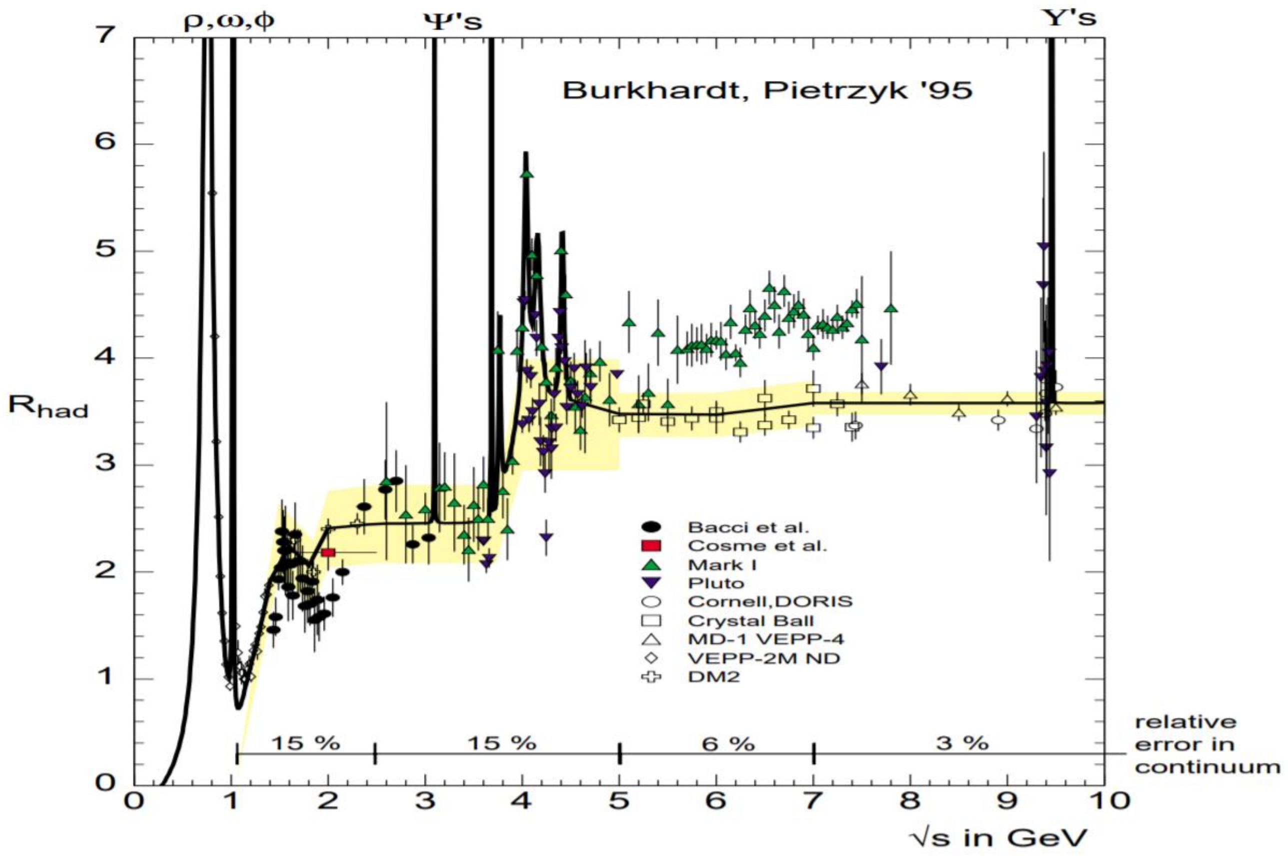
R measurements



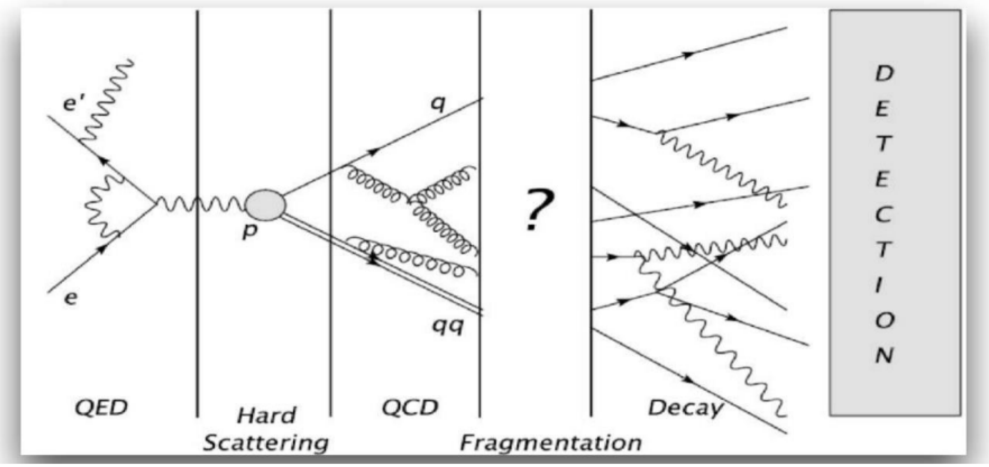


R measurements



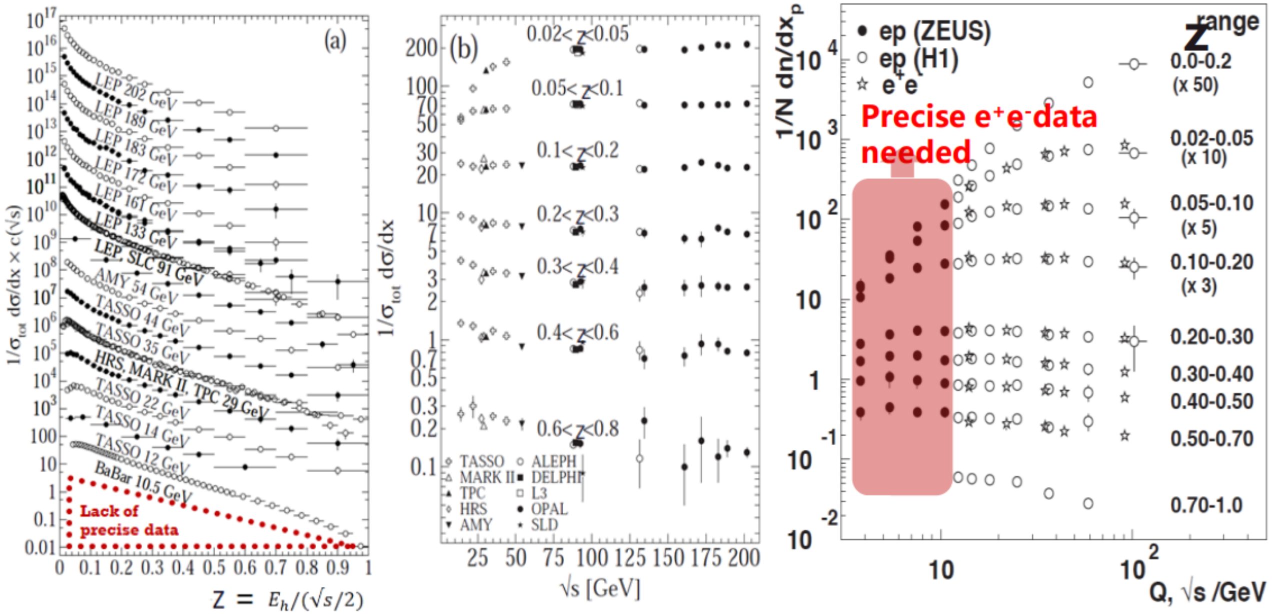


Fragmentation functions

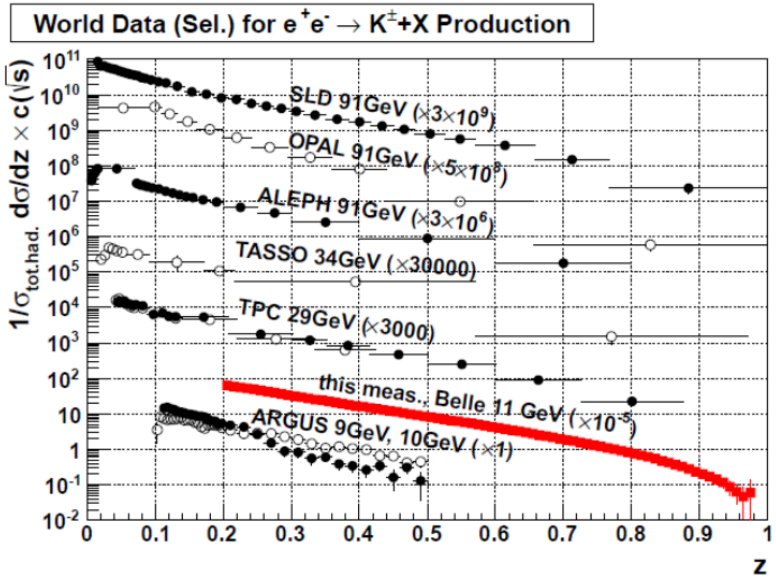


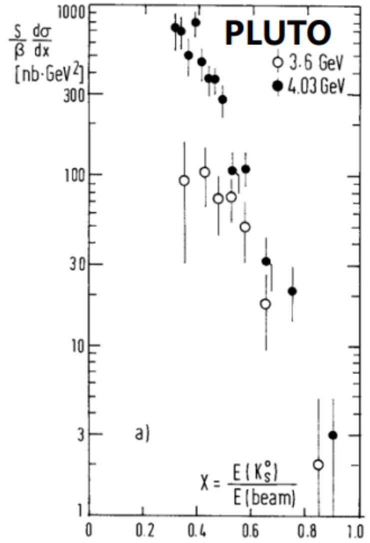
Fragmentation function $D_q^h(z)$: probability that hadron h is found in the debris of a hadron carrying a fraction $z=2E_h/\sqrt{s}$ of parton's momentum.

World data: Pion



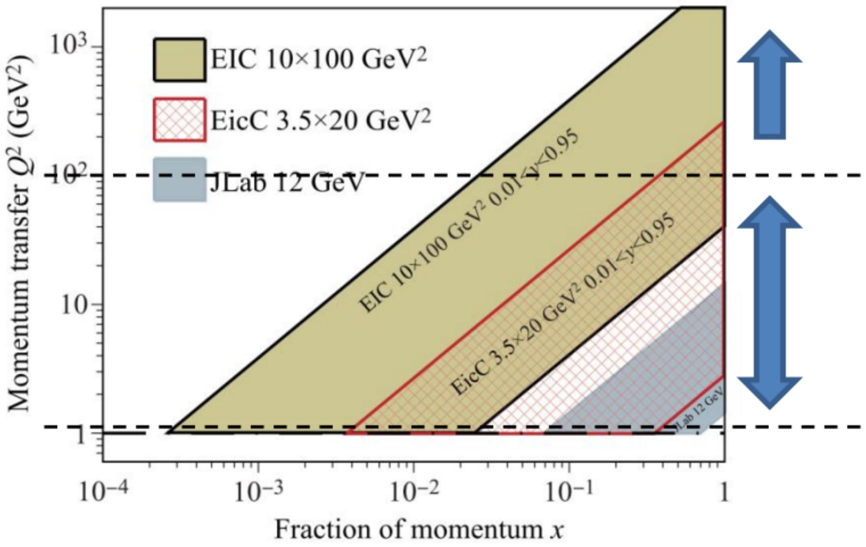
World data: Kaon

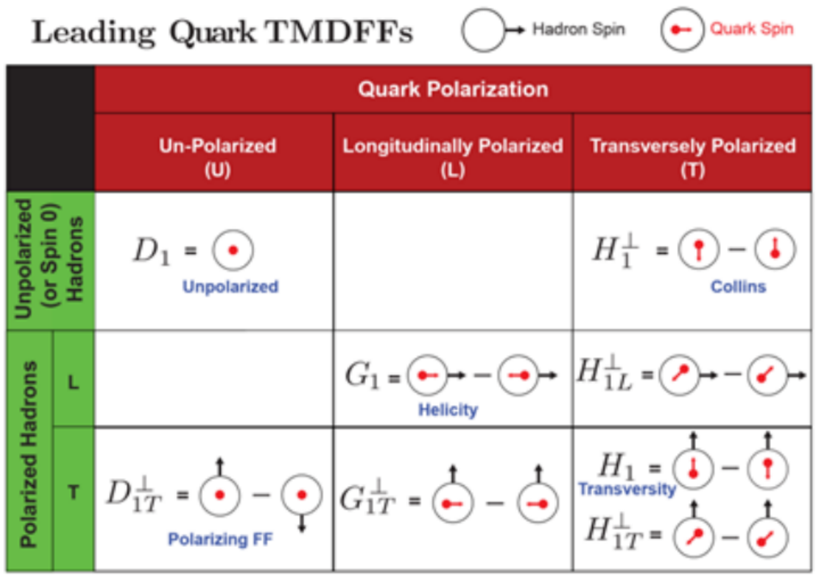




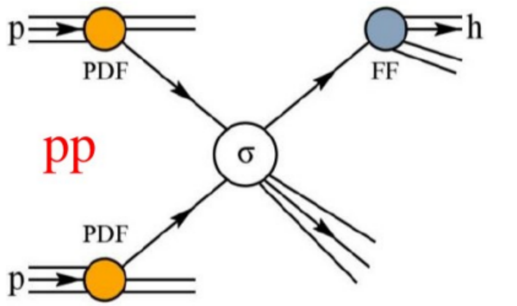
Fragmentation functions at STCF

- e^+e^- collider experiment provides the cleanest input for fragmentation functions (FFs) fitting. To accurately extract Parton Distribution Functions (PDFs), more precise FFs are required.
- Two types of FFs can be studied at an unpolarized e^+e^- collider: D and $H_1^{\perp}$. Multi-dimensional binning of the measurements can be provided.
- With polarized electron beam, more FFs can be studied.



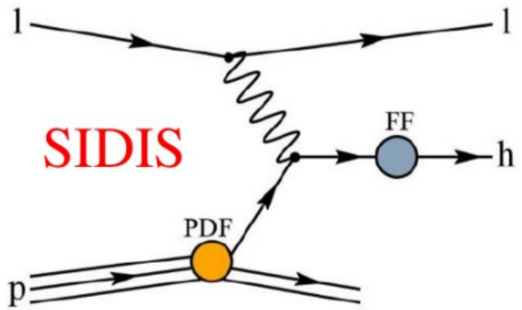


Access to Fragmentation Functions in Experiment



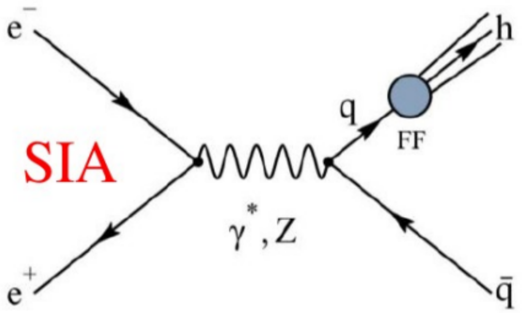
pp : $\sigma = \sum_q PDF \otimes PDF \otimes \sigma(q_1 q_2 \to q_1' q_2') \otimes FF$

- Depends on unpolarized PDFs
- Parton momenta not directly known
- Leading access to gluon FF



SIDIS : $\sigma = \sum_q PDF \otimes \sigma(eq \to e'q') \otimes FF$

- Depends on unpolarized PDFs
- Flavor structure directly accessible



e^+e^- : $\sigma = \sum_q \sigma(e^+e^- \to q\bar{q}) \otimes FF$

- PDFs not involved
- Calculations known at NNLO
- Flavor structure not directly accessible
- Cleanest access to Fragmentation Functions

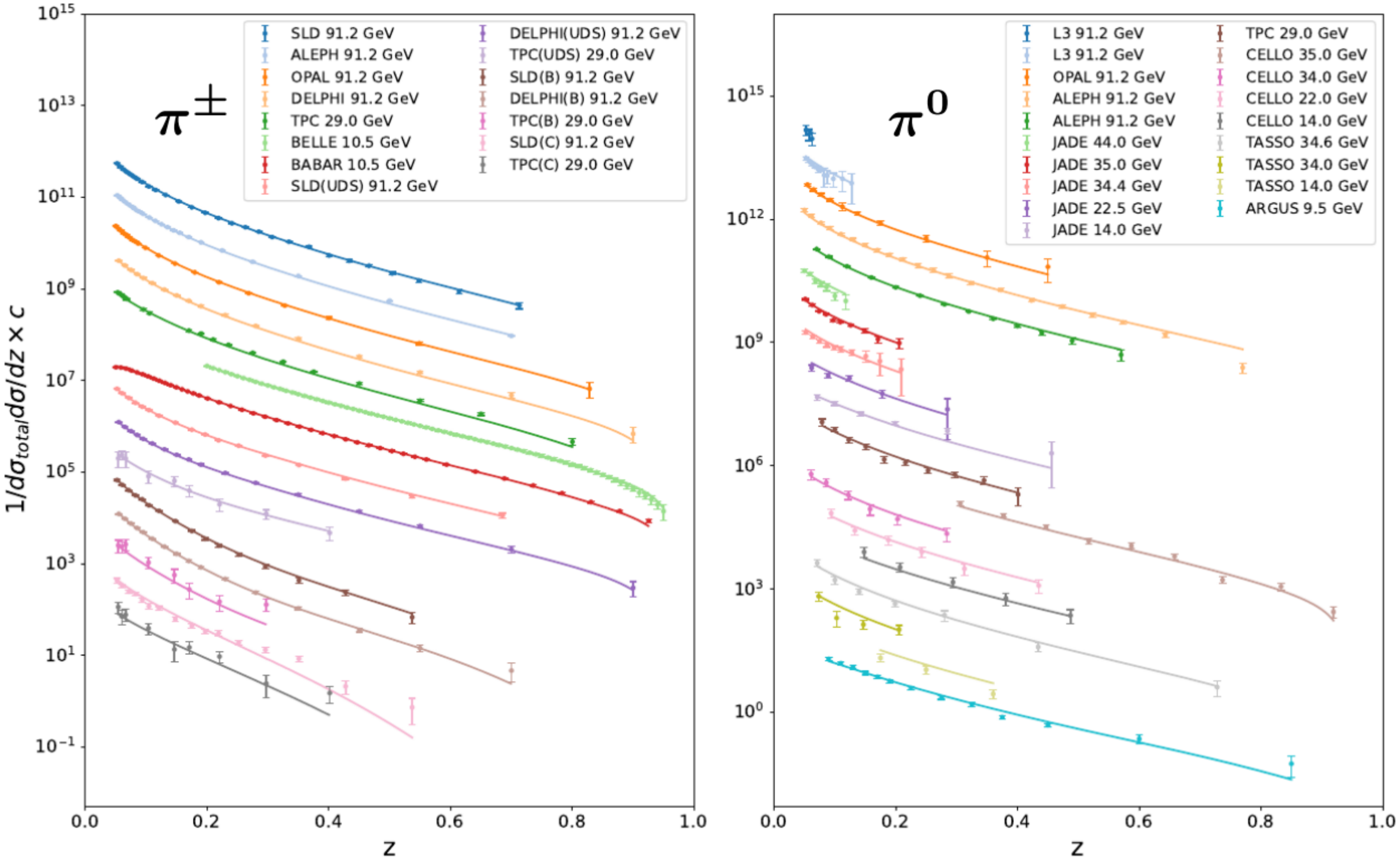
Experimental observable:

$$\frac{1}{\sigma_{\text{had,tot}}} \frac{d\sigma_{\text{h}}}{dz}$$

At leading order:

$$e^+e^- \to \text{h}X \sim \sum_q e_q^2 D_1^{h/q}(z)$$

Available World Data



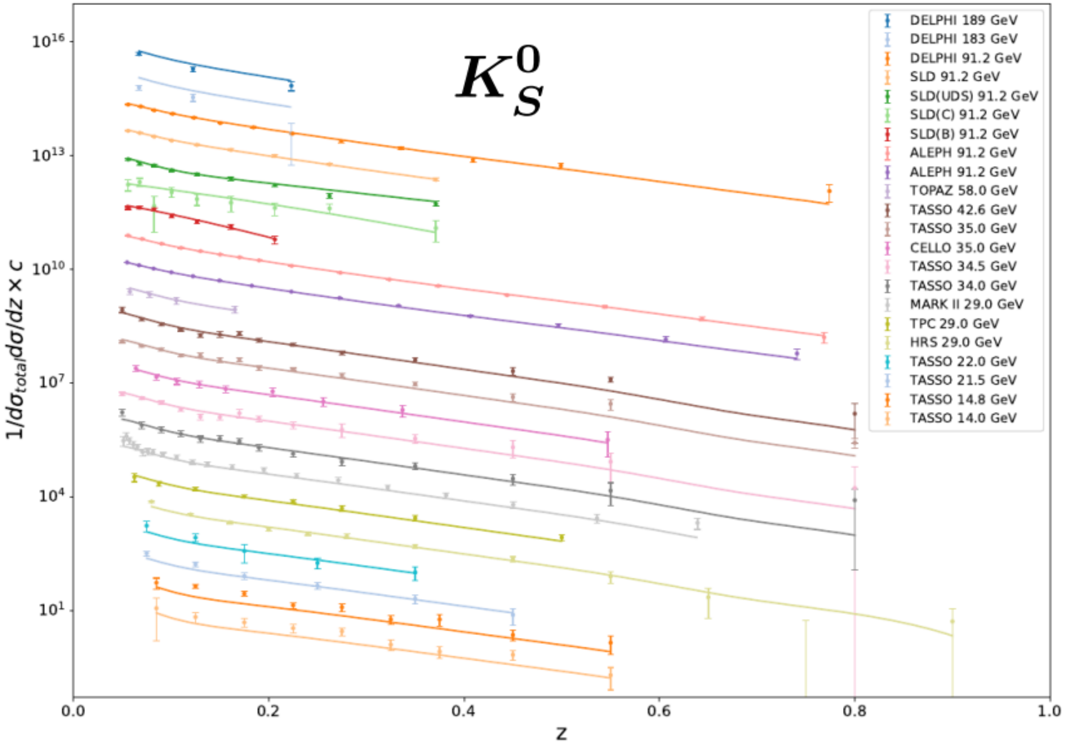
Li, Anderle, Xing, Zhao
arXiv:2404.11527

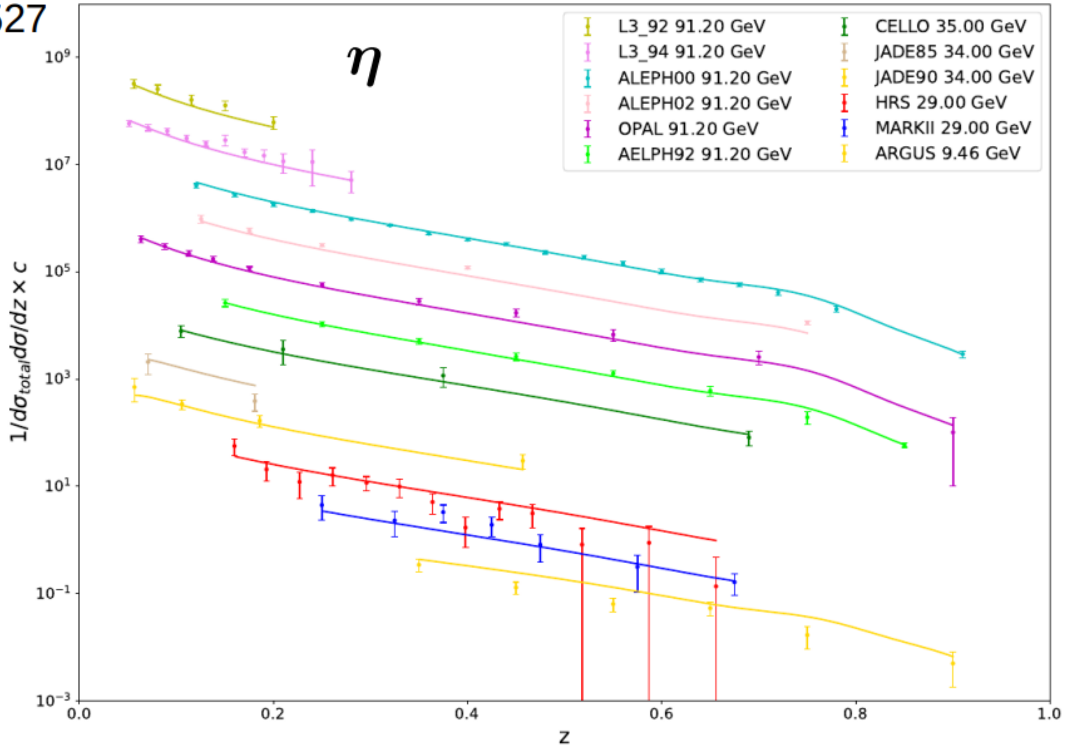
- Most information at high energies (SLAC, CERN, DESY)
- Lack of data below 10 GeV
- Unique opportunity for BESIII:

$$2 \leq \sqrt{s}\,[\text{GeV}] \leq 5$$

Available World Data

Li, Anderle, Xing, Zhao
arXiv:2404.11527





Lack of precise data at low energies, where BESIII can contribute!

and VEPP-6

Unpolarized FFs measurements at BESIII

Experimental observable at e^+e^- colliders:

$$\frac{1}{\sigma_{tot}(e^+e^- \to hadrons)} \frac{d\sigma(e^+ e^- \to h + X)}{d P_h}$$

h is a particular type of hadron such as π^0, $\pi^{+/-}$, $K^{+/-}$ …

- At Leading order $\sim \sum_q e_q^2 D_1^{h/q}(z)$

Unpolarized fragmentation function (D)

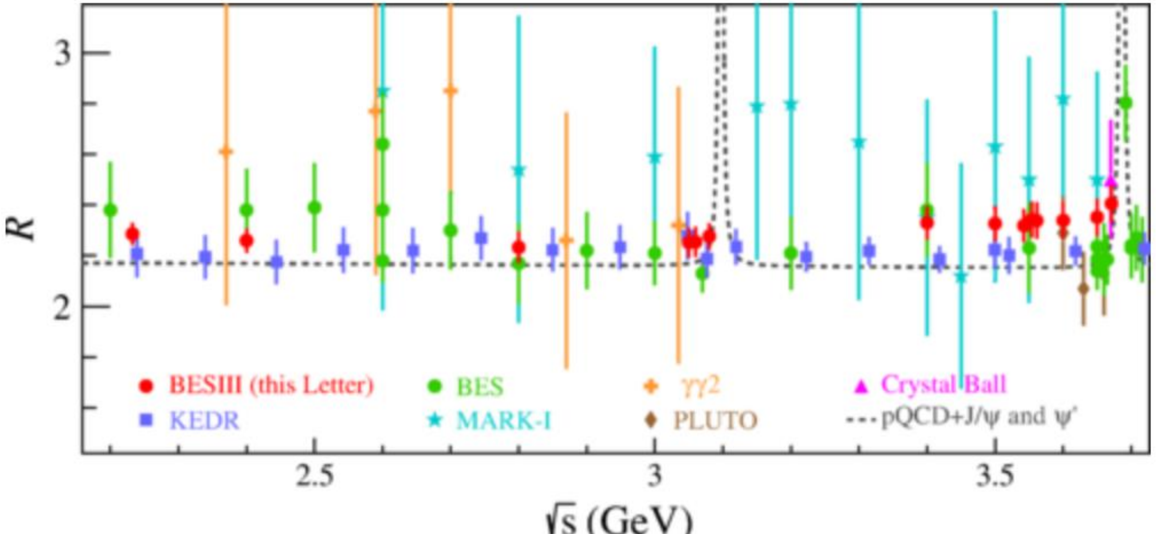
Fractional energy of hadron $z = 2E_h/\sqrt{s}$

Normalized Hadronic Cross Section

$$\frac{1}{\sigma_{\text{had}}} \cdot \frac{d\sigma(e^+e^- \to h + X)}{dp_h} = \frac{N_h}{N_{\text{had}}} \frac{1}{\Delta p_h} = \frac{N_h{}^{\text{obs}}}{N_{\text{had}}^{\text{obs}}} \frac{1}{\Delta p_h} f_h$$

Inclusive hadronic cross section

- R-Value measurement at BESIII

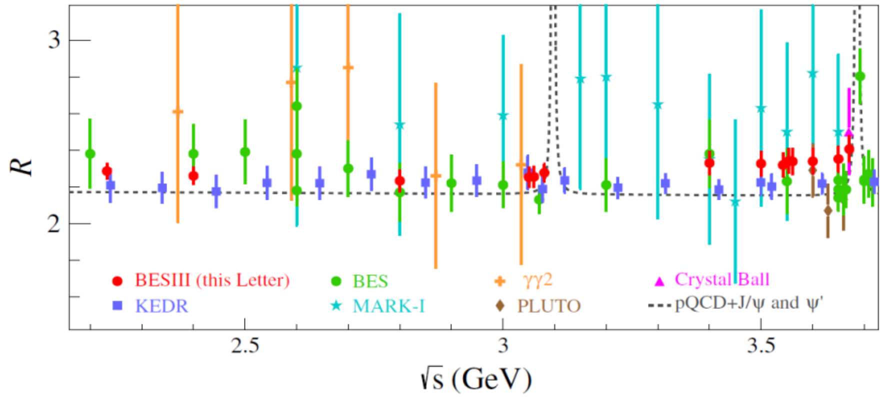


Differential inclusive production cross section of hadron *h*

Correction factor

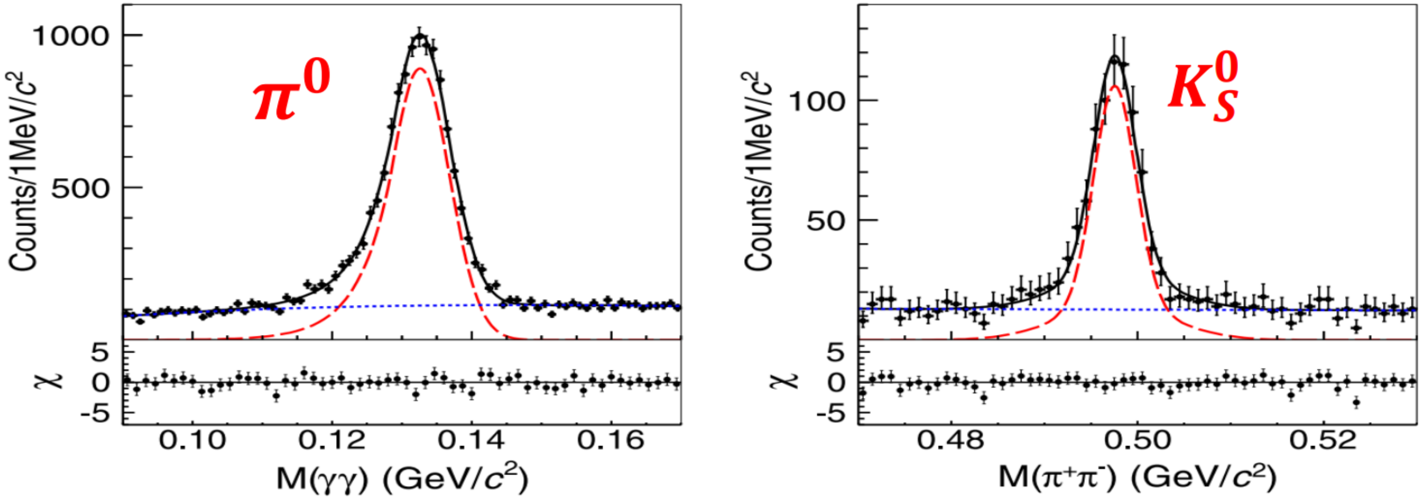
- Reconstruction efficiency
- Radiative corrections
- Based on generator development for R-Value measurement

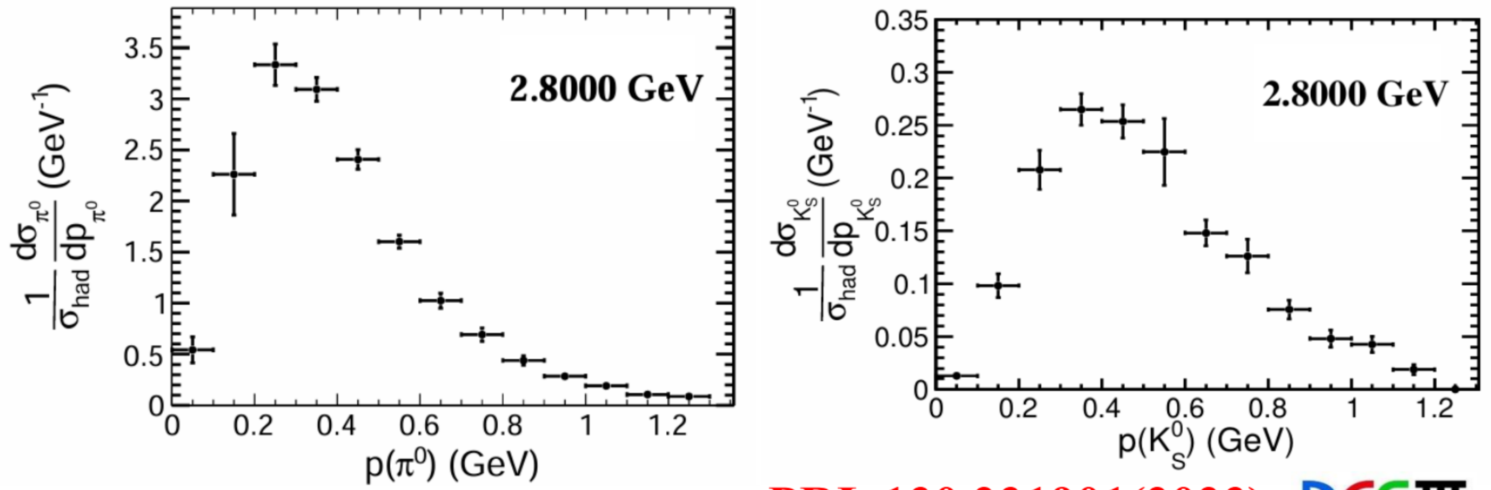
Inclusive π^0/K_S^0 production



$\sqrt{s}$ (GeV)	$\mathcal{L}$ (pb^{-1})	$N_{\rm had}^{\rm tot}$	$N_{\rm bkg}$
2.2324	2.645	83227	2041
2.4000	3.415	96627	2331
2.8000	3.753	83802	2075
3.0500	14.89	283822	7719
3.4000	1.733	32202	843
3.6710	4.628	75253	6461

$p \in (0.4, 0.5)$ GeV

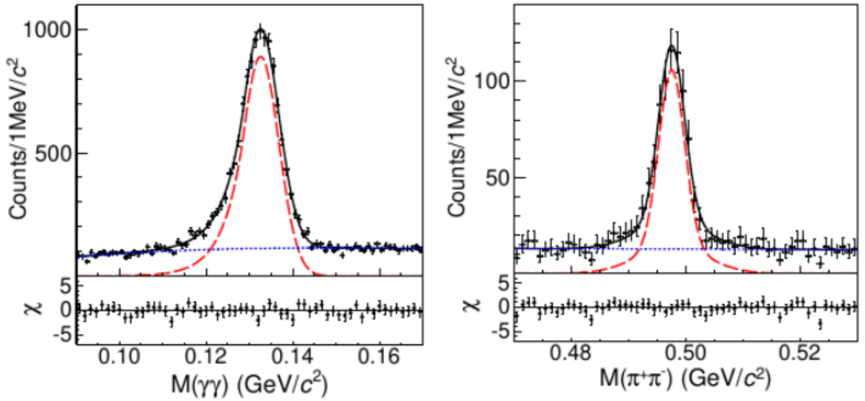




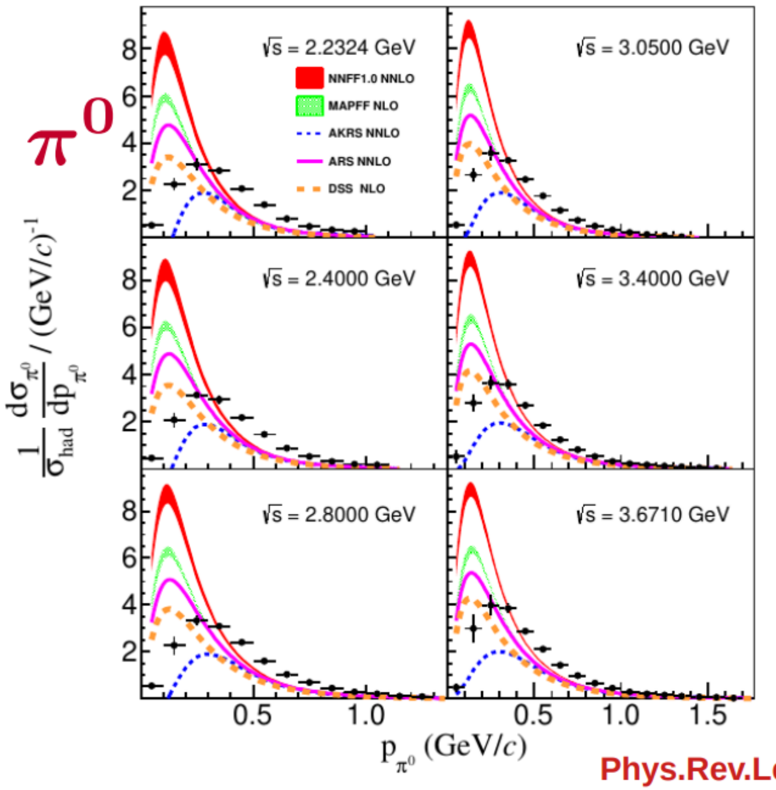
PRL 130 231901(2023)

Results for π^0 and K_S^0

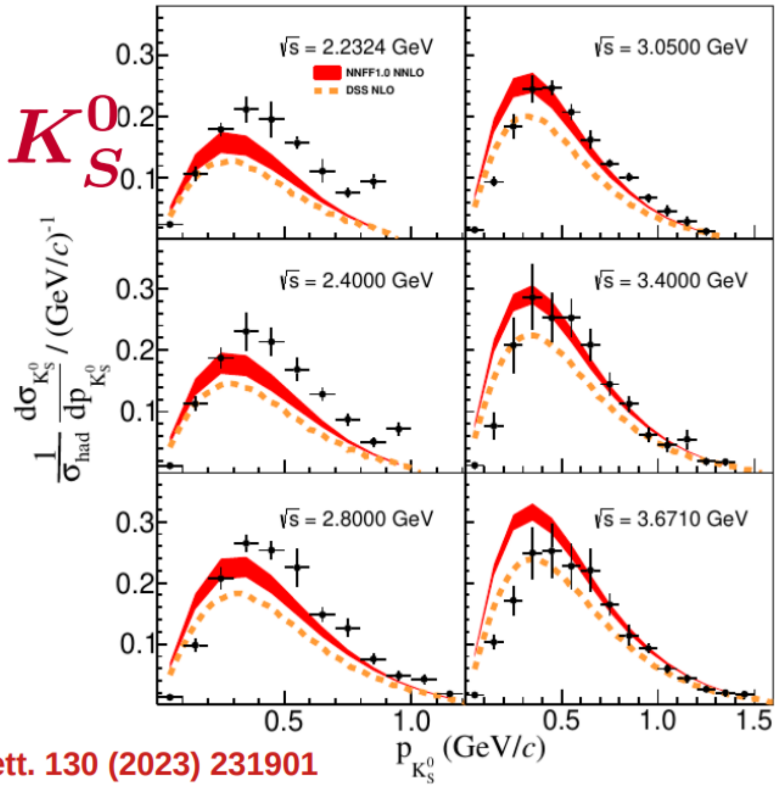
$p \in (0.4, 0.5)$ GeV/c



- Hadrons reconstructed from daughters
- Background suppression:
  - Helicity angle cut
  - Secondary vertex fit

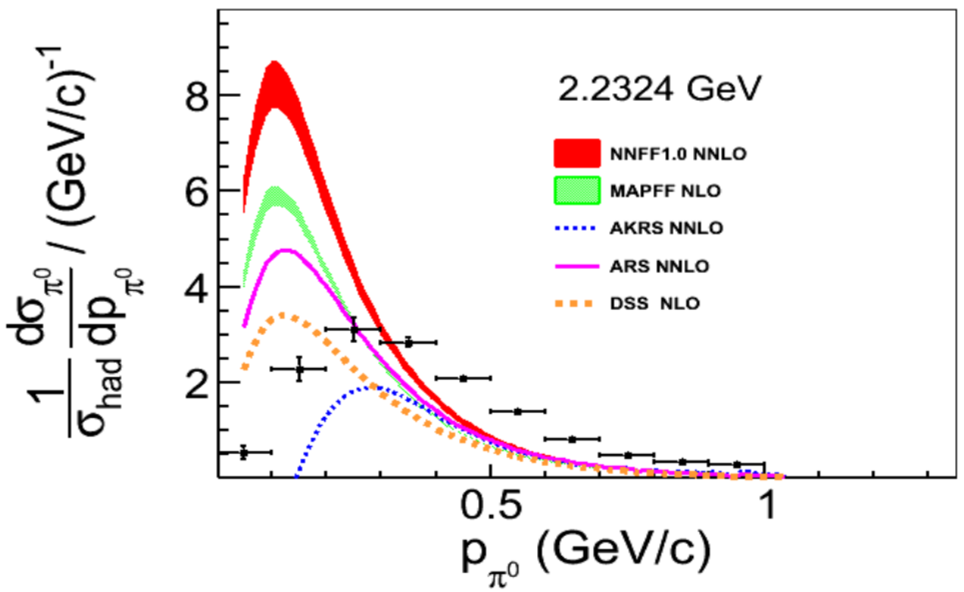


Phys.Rev.Lett. 130 (2023) 231901



- Disagreement with existing fits of Fragmentation Functions
  - Depending on $\sqrt{s}$ and p_h
  - Problem in extrapolation of Fragmentation Functions to lower energies?

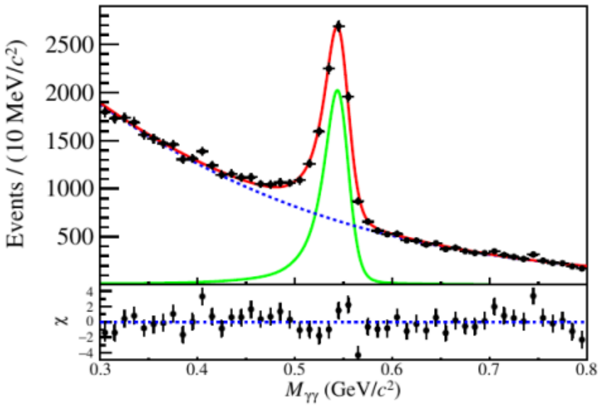
Results: inclusive π^0/K_s^0



PRL 130 231901(2023)

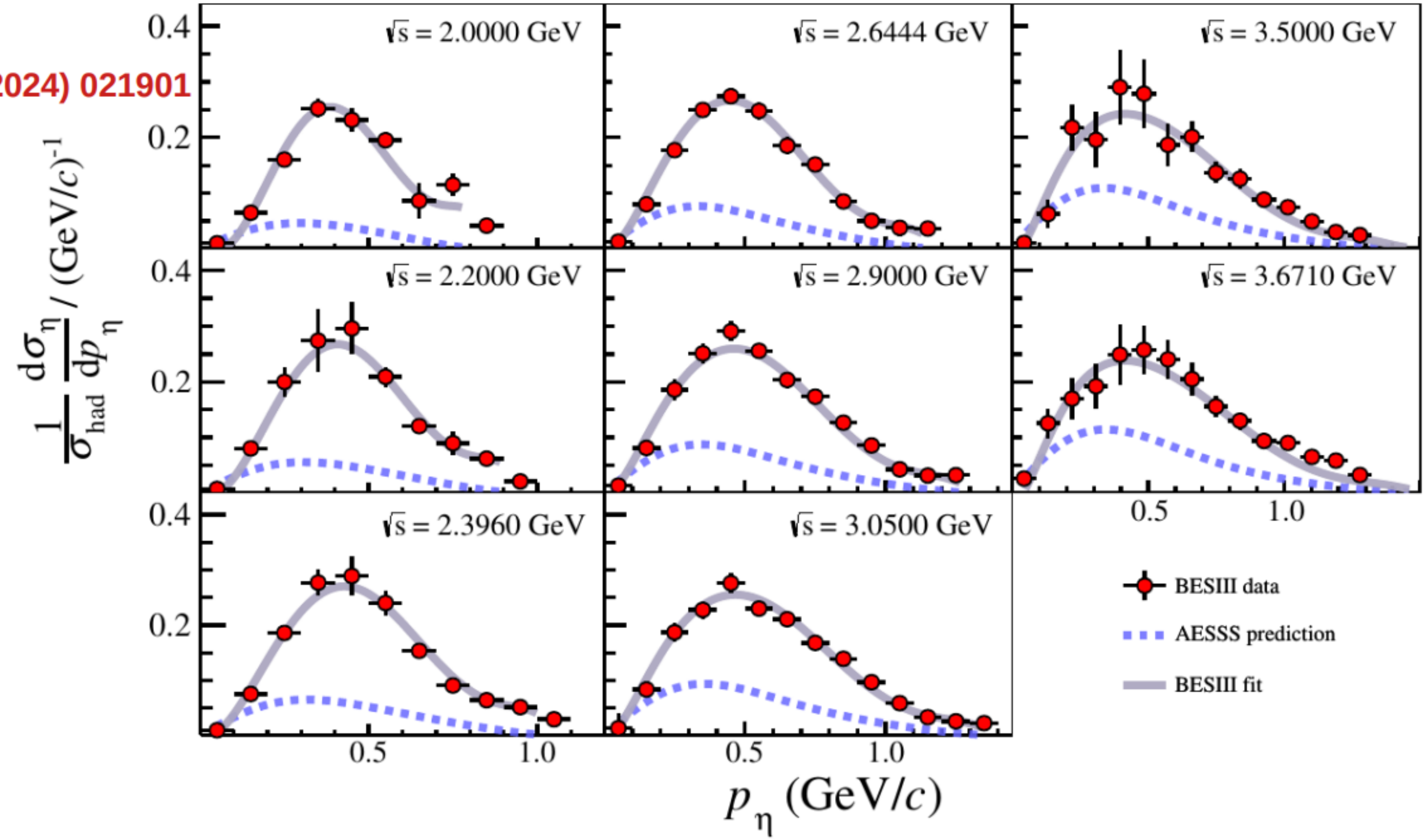
- From theory side: fitting with BESIII data, hadron
- mass effect, large z re-summation, and so on
- From experimental side
  - Primary hadron vs from resonance decay
  - $\Longrightarrow$ measure $e^+ e^- \rightarrow \rho(\omega, \phi) + X$, and so on
  - Contribution of vector states ρ^*, ω^* and ϕ^*
  - $\Longrightarrow e^+ e^- \rightarrow \rho^*/\omega^*/\phi^* \rightarrow h + X$

Results for η



(0.4, 0.5) GeV/c @ $\sqrt{s}$ = 2.9 GeV

Phys.Rev.Lett. 133 (2024) 021901



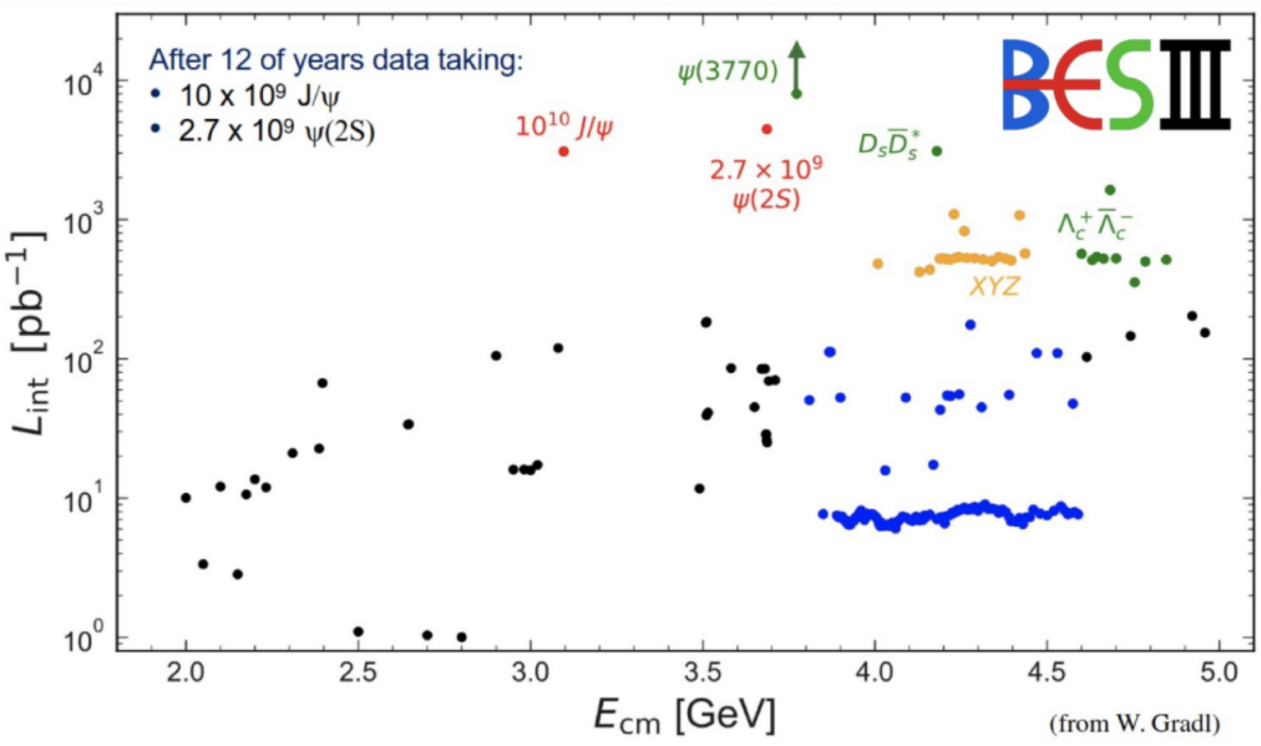
- Hadrons reconstructed from daughters
- Helicity angle cut for background suppression

- Disagreement with fit of Fragmentation Functions in Phys.Rev. D83 (2011) 034002
- Agreement with new fit by Li, Anderle, Xiao, Zhang (arXiv:2404.11527)
  - Includes NNLO accuracy, higher-twist effects, and hadron mass correction

Further Measurements at BESIII

Large amounts of additional data already collected

- 170 energy scan points with $>10^5$ hadrons



Continuum region (2.00 – 3.67 GeV):

- Inclusive production of charged particles (p vs. p_t)

$$e^+e^- \to \pi^\pm, K^\pm + X$$

- Spin-alignment effects of vector mesons

$$e^+e^- \to \phi, K^* + X$$

Higher energies

- Access to heavier mesons and hyperons

$$e^+e^- \to \eta', \Lambda/\Sigma + X$$